Practical round. Sketches for solutions. For jury job ONLY

Note. The given sketches are not full; the team leaders have to give more detailed explanations to students. But the correct solutions in the students' papers (enough for 10 pts) may be shorter (and without any word, of course).

Note. Jury members should evaluate the student's solutions in essence, and not by looking on formal existence the mentioned sentences or formulae. The formal presence of the mentioned positions in the text is not necessary to give the respective points. Points should be done if the following steps de facto using these positions.

αβ-7. Analemma.

(7.0. preliminary notes and calculations) At first we may calculate the scale of photo.

Horizontal scale: 201 mm / 60° = 3.35 mm/°.

Vertical scale: 288 mm / 86° ≈ 3.35 mm/°.

So the scale is the same on horizontal and vertical axis.

As the photo was taken in East direction, we should assume that the prime vertical runs exactly in the middle of the page.

We may find approximately the horizon line by the reference landscape right to the mountain (to use position left to the mountain is not very good as there are some hills in that direction.

The center of "mean" Sun rises 6^H00^M just at the point of East on the horizon at moments of equinoxes. So we may determine this point (crossing point of horizon and the prime vertical) as zero point for our drawings.

The position of the Sun depending on date is:

$$h = \delta\sin\varphi - \eta\cos\varphi \quad \text{(axis direction up)},$$

$$A = -\eta\sin\varphi \quad \text{(axis direction right (South))}.$$

$\varphi = 46°56'$ and δ and η depending on date we may find in the ephemeris table.

(7.1.) So, calculating with this formulae the students have to fill a table:

Date	h	A

(7.2.) The equator passes through our zero point at an angle $\beta = 90° - \varphi = 43°04'$ to the horizon, as it shown in the "picture of solution".

(7.3.) Using the calculated values, we may get approximately the following result (see background of this page, the scale fit to printing this page on A4 page, but the actual dates are different in this example). The analemma in the landscape of the actual picture is presented in separate answer page.

(7.4.) Time of local time zone ahead the mean solar time by the value

$$(45° - 26°22')/(15°/h) \approx 1^H14.5^M.$$

So an analemma we should take the pictures at $6^H00^M + 1^H14.5^M = 7^H14.5^M$ to get such an analemma.

(7.5.) Today's position of the Sun is pointed by thick filled circle.

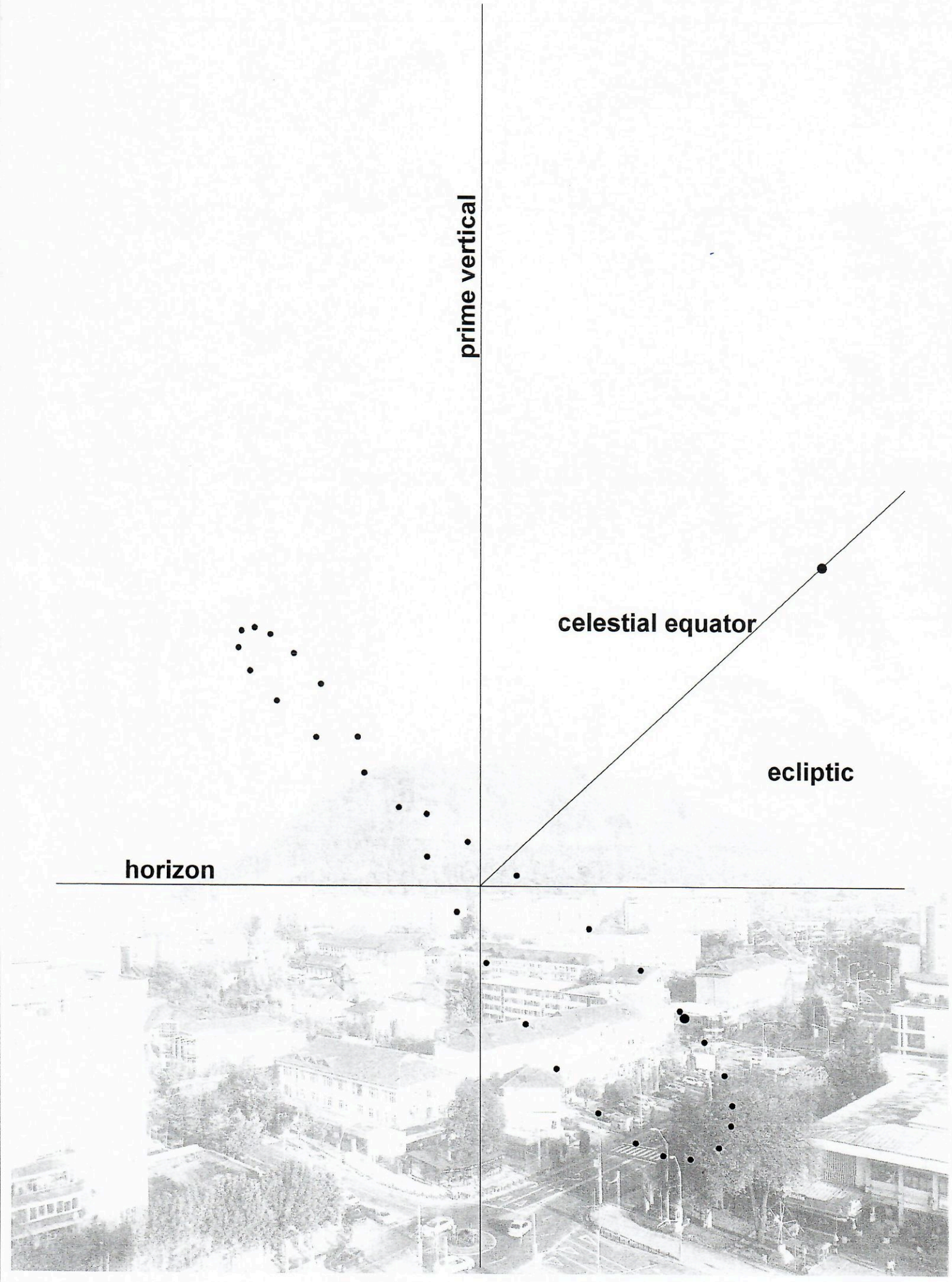
(7.6.) The ecliptic passes today's position of the Sun and the point of autumn equinox at the celestial equator, which is in ~31° (as duration from September 23 till October 24) right-up along the celestial equator from the zero point, L = 104 mm in our scale,

$$\Delta X = L\cos\varphi = 71 \text{ mm},$$

$$\Delta Y = L\sin\varphi = 76 \text{ mm}.$$

This point is marked also by thick filled circle. The line of ecliptic is shown in answer page with analemma. Sorry at the current version I cannot scan the picture, in the enclosed picture the line of ecliptic should be drawn by hand crossing the "thick" images of the Sun.

Marking scheme: proposed to be elaborated by the actual Jury members. Total number of points is 10 for each of two problems of the Practical round.


prime vertical
celestial equator
ecliptic
horizon

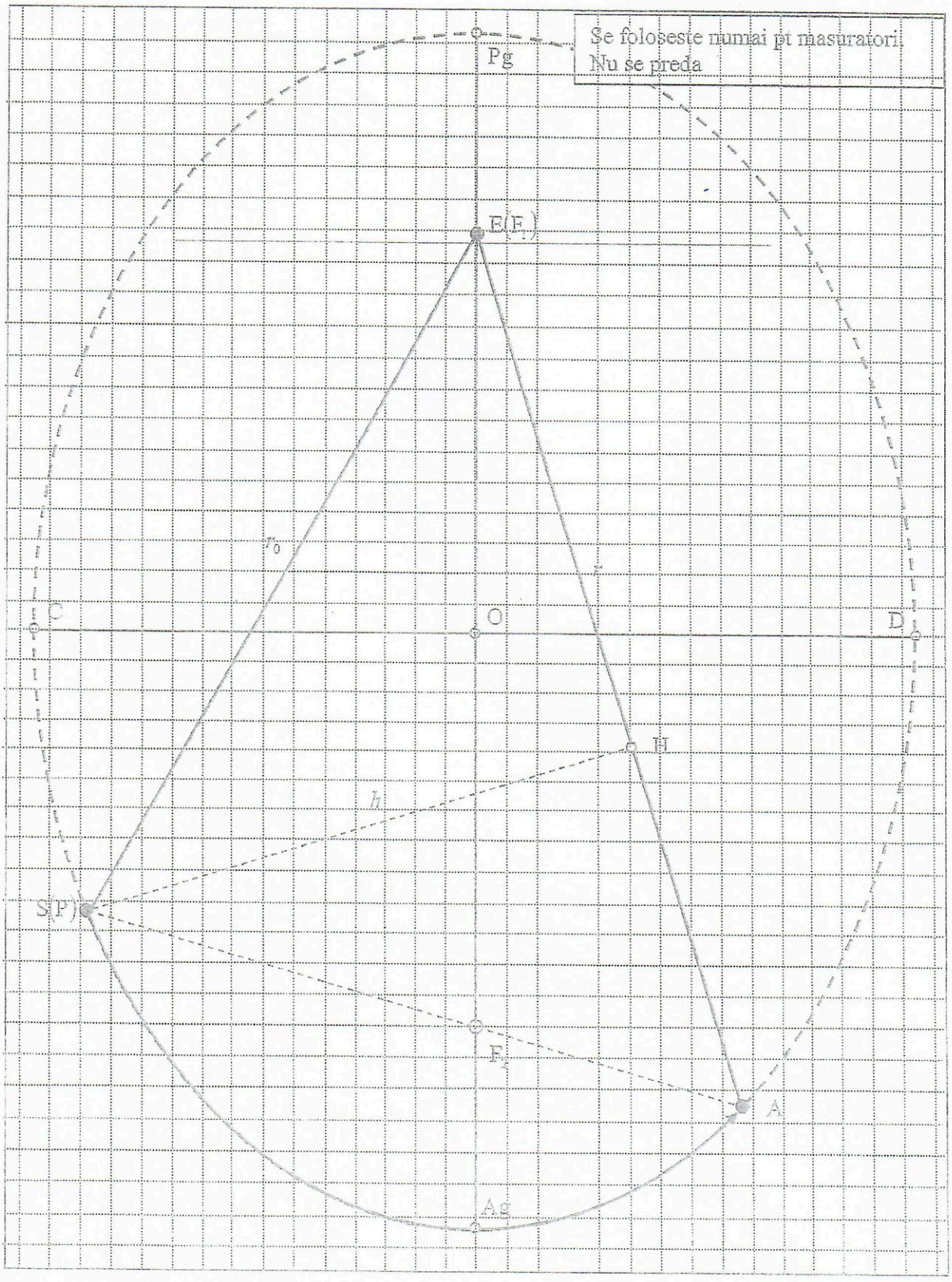

Se foloseste numai pt masuratori
Nu se preda
Pg
E(F1)
C
O
D
H
S(P)
F1
A
Ag

Fig. 1

Rezolvare

$\alpha\beta.1-1.$

Pentru stabilirea elementelor orbitei eliptice a proiectilului (semiaxa mare, a; semiaxa mică, b; excentricitatea, e; distanța minimă, r_{min}; distanța maximă, r_{max}), știind că:

$$r = 30010{,}88\ \text{km};\ r_{\text{masurat desen}} = 153\ \text{mm},$$

rezultă că scara tuturor dimensiunilor geometrice liniare, în desenul din figura 1 este:

$$S = \frac{r}{r_{\text{masurat desen}}} = \frac{30010{,}88}{153}\ \frac{\text{km}}{\text{mm}},$$

astfel încât obținem:

$$r_{0,\text{masurat}} = 133\ \text{mm};\ r_0 = r_{0,\text{masurat desen}} \cdot S = 26087{,}88\ \text{km};$$

$$r_{\text{masurat}} = 153\ \text{mm};$$

$$2a_{\text{masurat desen}} = 200\ \text{mm};\ a_{\text{masurat desen}} = 100\ \text{mm};$$

$$a = a_{\text{masurat desen}} \cdot S \approx 19614{,}95\ \text{km};$$

$$2b_{\text{masurat desen}} = 148\ \text{mm};\ b_{\text{masurat desen}} = 74\ \text{mm};$$

$$b = b_{\text{masurat desen}} \cdot S \approx 14515{,}06\ \text{km};$$

$$2c_{\text{masurat desen}} = 134\ \text{mm};\ c_{\text{masurat desen}} = 67\ \text{mm};$$

$$c = c_{\text{masurat desen}} \cdot S \approx 13142{,}01\ \text{km};$$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{b^2_{\text{masurat desen}}}{a^2_{\text{masurat desen}}}} \approx 0{,}67;$$

$$r_{\text{min, masurat desen}} = 33\ \text{mm};$$

$$r_{\text{min}} = r_{\text{min, masurat desen}} \cdot S \approx 6472{,}93\ \text{km};\ r_{\text{min}} = a(1-e);$$

$$r_{\text{max, masurat desen}} = 167\ \text{mm};$$

$$r_{\text{max}} = r_{\text{max, masurat}} \cdot S \approx 32756{,}97\ \text{km};\ r_{\text{max}} = a(1+e).$$

$\alpha\beta.1-2.$

$$T^2 = \frac{4\pi^2}{G(M+m)} \cdot a^3;\ m \ll M;\ T^2 = \frac{4\pi^2}{GM} \cdot a^3;$$

$$M = 5{,}972 \cdot 10^{24}\ \text{kg};\ G = 6{,}674 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2};\ \pi = 3{,}14159265;$$

$$a = 19614{,}95\ \text{km};$$

$$T = 2\pi\sqrt{\frac{a^3}{GM}} = 2 \cdot 3{,}14159265 \cdot \sqrt{\frac{19614{,}95^3 \cdot 10^9}{6{,}674 \cdot 10^{-11} \cdot 5{,}972 \cdot 10^{24}}}\ \text{s} = 27340{,}57\ \text{s};$$

$$T = 455{,}67\ \text{min} = 7{,}59\ \text{h}.$$

$\alpha\beta.1-3.$

Pe elipsa de transfer din figura 1 se construiește segmentul $SH \perp EA$, reprezentând înălțimea triunghiului ESA și se măsoară pe desen lungimea acestui segment:

$$(SH)_{\text{masurat desen}} = h_{\text{masurat desen}} = 98\ \text{mm}.$$

$$\Sigma_{\text{elipsa transfer}} = \pi ab = \pi S a_{\text{masurat desen}} \cdot S b_{\text{masurat desen}} = \pi S^2 a_{\text{masurat desen}} b_{\text{masurat desen}};$$

$$\Sigma_{\text{sector elipsa}} = \Sigma_{\Delta(ESA)} + \Sigma_{\text{segment elipsa}};$$

$$\Sigma_{\Delta(ESA)} = \frac{1}{2}rh = \frac{1}{2}Sr_{\text{masurat desen}}Sh_{\text{masurat desen}} = \frac{1}{2}S^2 r_{\text{masurat desen}} h_{\text{masurat desen}}.$$

Pentru calculul lui $\Sigma_{\text{segment elipsa}}$, prin numărare și întregire, apreciem că acolo există $n = 100$ mici pătrate, fiecare cu lungimea laturii $l_{\text{masurat desen}} = 5\,\text{mm}$, astfel încât, corespunzător aceleeași scări, S, rezultă:

$$\Sigma_{\text{segment elipsa transfer}} = n \cdot Sl_{\text{masurat desen}} \cdot Sl_{\text{masurat desen}} = nS^2 l^2_{\text{masurat desen}}.$$

În aceste condiții, rezultă:

$$\Sigma_{\text{sector elipsa transfer}} = \Sigma_{\Delta(ESA)} + \Sigma_{\text{segment elipsa}};$$

$$\Sigma_{\text{sector elipsa transfer}} = \frac{1}{2}S^2 r_{\text{masurat desen}} h_{\text{masurat desen}} + nS^2 l^2_{\text{masurat desen}}.$$

În acord cu legea a doua a lui Kepler (legea ariilor), rezultă:

$$T \ldots\ldots \Sigma_{\text{elipsa transfer}};$$

$$\Delta t \ldots\ldots \Sigma_{\text{sector elipsa transfer}};$$

$$\Delta t = \frac{\Sigma_{\text{sector elipsa transfer}}}{\Sigma_{\text{elipsa transfer}}} \cdot T,$$

reprezentând durata deplasării proiectilului P până la întâlnirea cu asteroidul A.

În aceste condiții, rezultă:

$$\Delta t = \frac{\frac{1}{2}S^2 r_{\text{masurat desen}} h_{\text{masurat desen}} + nS^2 l^2_{\text{masurat desen}}}{\pi S^2 a_{\text{masurat desen}} b_{\text{masurat deseb}}} \cdot T;$$

$$\Delta t = \frac{\frac{1}{2}r_{\text{masurat desen}} h_{\text{masurat desen}} + nl^2_{\text{masurat desen}}}{\pi a_{\text{masurat desen}} b_{\text{masurat desen}}} \cdot T;$$

$$r_{\text{masurat desen}} = 155\,\text{mm};\ h_{\text{masurat desen}} = 98\,\text{mm};\ l_{\text{masurat desen}} = 5\,\text{mm};$$

$$n = 100;\ a_{\text{masurat desen}} = 100\,\text{mm};\ b_{\text{masurat desen}} = 74\,\text{mm};$$

$$T = 455{,}67\,\text{min} = 7{,}59\,\text{h};$$

$$\pi = 3{,}14159265;$$

$$\Delta t = \frac{\frac{1}{2}155\,\text{mm} \cdot 98\,\text{mm} + 100 \cdot 25\,\text{mm}^2}{3{,}14159265 \cdot 100\,\text{mm} \cdot 74\,\text{mm}} \cdot 7{,}59\,\text{h};$$

$$\Delta t = \frac{155 \cdot 49 + 100 \cdot 25}{3{,}14159265 \cdot 100 \cdot 74} \cdot 7{,}59\,\text{h};$$

$$\Delta t = 3{,}29\,\text{h}.$$

α.1.

$$d_{\text{SA, masurat}} = 115\,\text{mm};$$

$$d_{\text{SA}} = d_{\text{SA, masurat desen}} \cdot S \approx 22549{,}01\,\text{km}.$$

Se modelează un fir de-a lungul sectorului elipsei cuprins între punctele S și A. Măsurând lungimea acestui fir, rezultă:

$$l_{\text{SA, masurat}} = 135\,\text{mm};$$

$$l_{\text{SA}} = l_{\text{SA, masurat}} \cdot S = 135\,\text{mm} \cdot \frac{30010{,}88\,\text{km}}{153\,\text{mm}} \approx 26480{,}18\,\text{km}.$$

§.1.

A determina direcția vitezei inițiale, $\vec{v}_0$, însemnează a stabili direcția tangentei la elipsă în punctul S. Aceasta se face folosind proprietatea optică a elipsei, conform căreia, razele de lumină plecate din focarul unei oglinzi concave elipsoidale, după reflexie trec toate prin celălalt focar al oglinzii, așa cum indică desenul din figura 3.

Ca urmare, direcția tangentei la elipsă, formează unghiuri egale cu razele vectoare ale lui S plecate din cele două focare, așa cum indică desenele din figurile 3, 4 și 5. Tangenta la elipsă în punctul S este perpendiculară pe bisectoarea unghiului ASE.

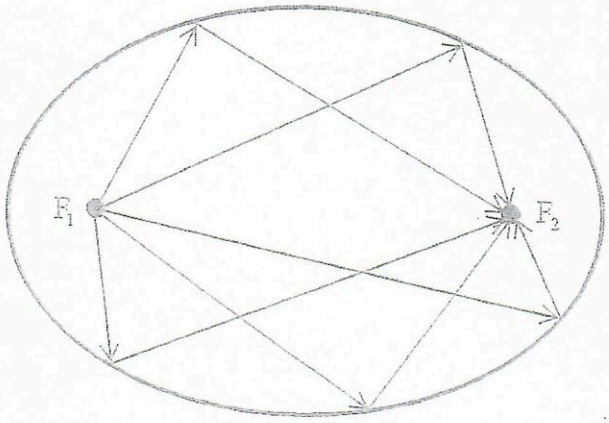


Fig. 3

În aceste condiții, direcția lui $\vec{v}_0$, formează cu direcția lui $\vec{r}_0$ unghiul α, care se măsoară direct pe desen cu ajutorul unui raportor, considerând că desenul a fost făcut, respectând valorile numerice ale mărimilor cunoscute: $r_0, r, \Delta\theta$.

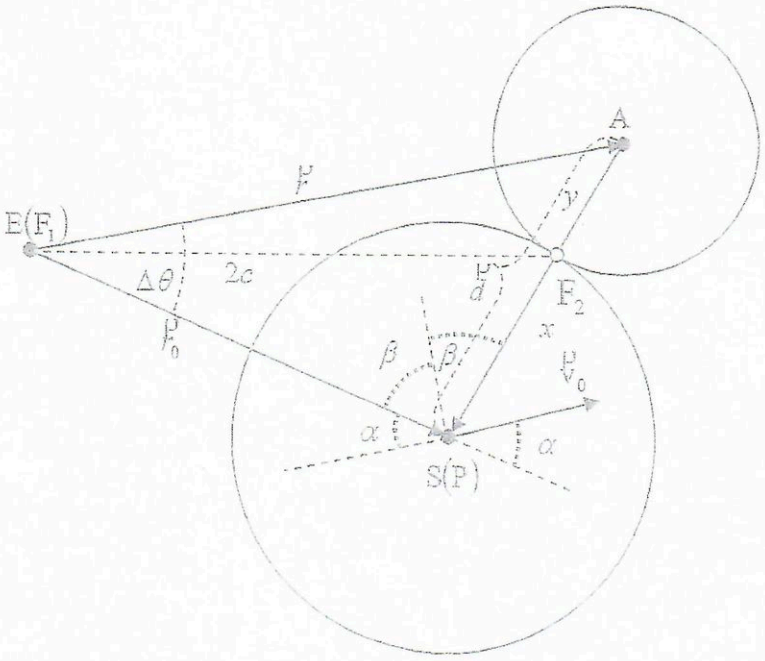


Fig. 4

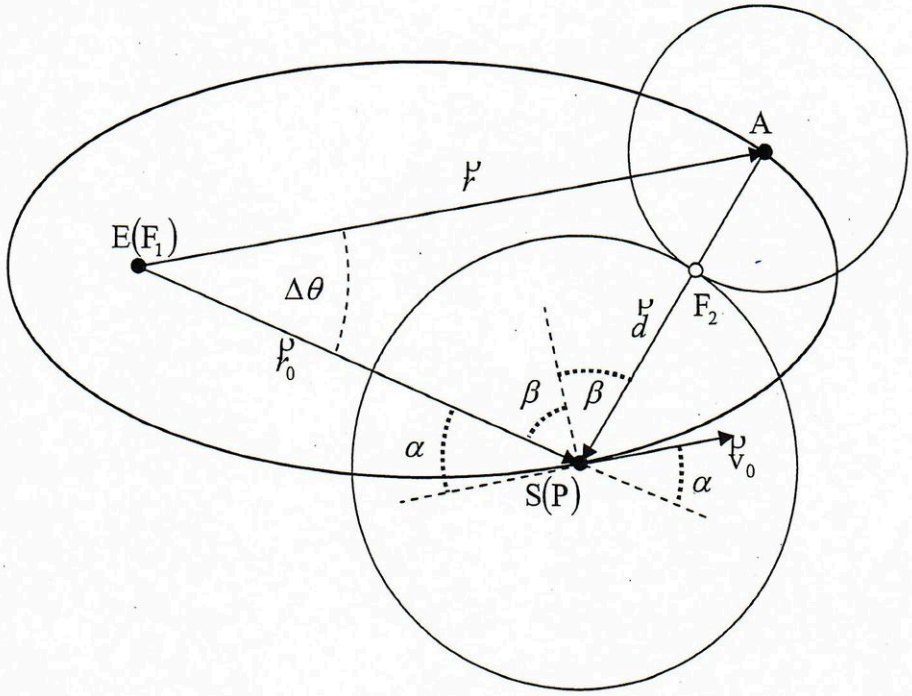


Fig. 5

În aceste condiții, direcția lui $\vec{v}_0$, corespunzătoare proiectilului P, lansat din punctul S, formează cu direcția lui $\vec{r}_0$ unghiul α, care poate fi măsurat în desenul din figura 6:

$$\alpha \approx 54^0.$$