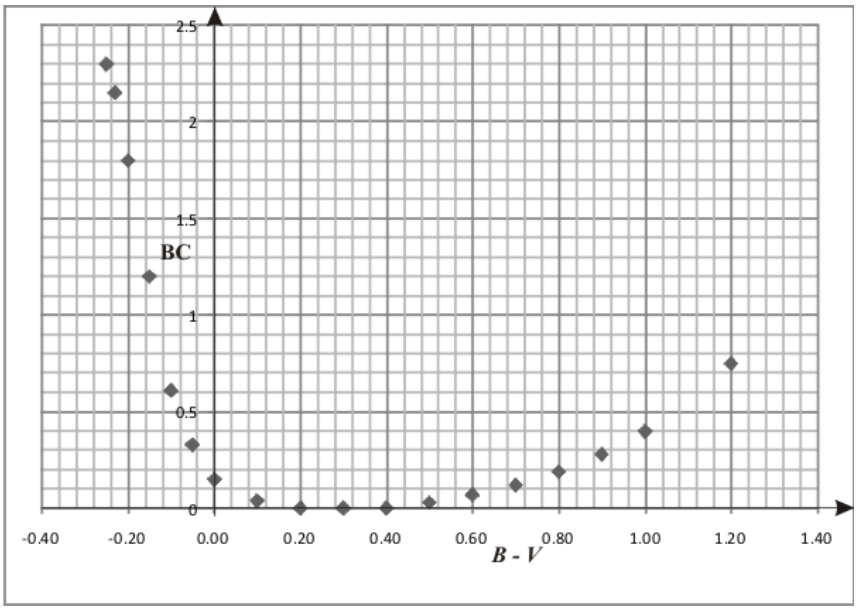
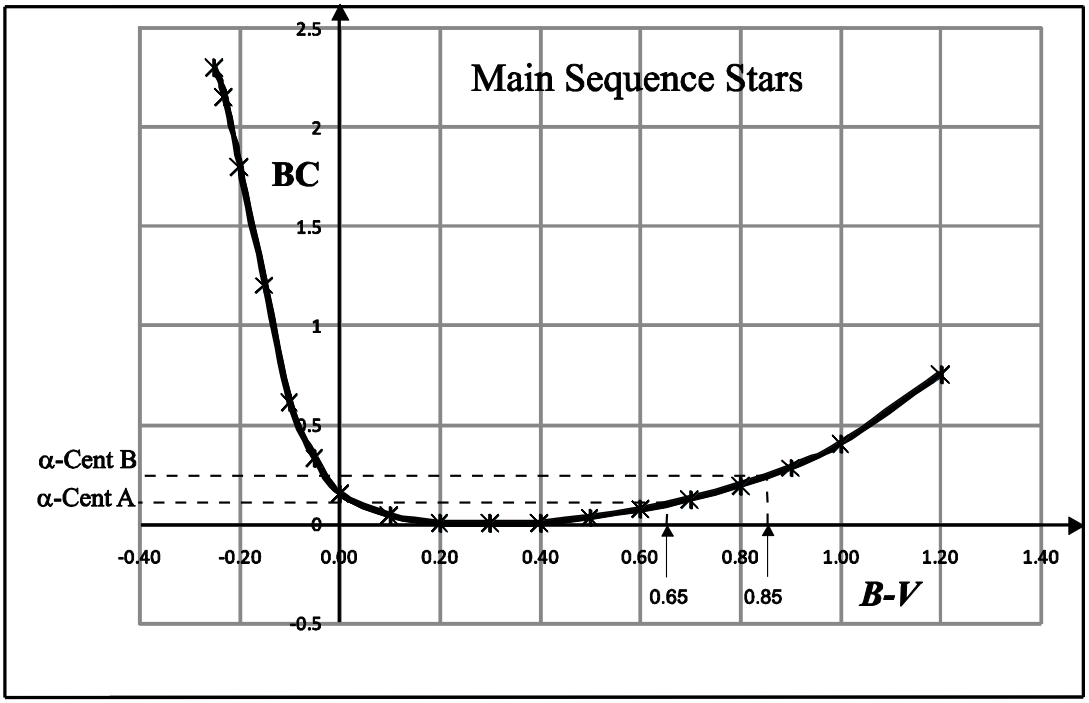
## II. Determination of stellar masses in a visual binary system

1




**(20%)**

2.




⊙ For α Cen. A. , its colour index is given by $B - V = 0.65$. Then, by taking the line that intersecting the above curve we find $BC = 0.10$. **(10%)**

⊙ For α Cen. B. , its colour index is given by $B - V = 0.85$. Then, by taking the

line that intersecting the above curve we find $BC = 0.25$. **(10%)**
From the relation $m_v - m_{bol} = BC$ it is then obtained,

⊙ For α Cen. A, $m_v = -0.01$ , we get $m_{bol} = -0.01 - 0.10 = -0.11$ **(10%)**

⊙ For α Cen. B, $m_v = 1.34$, we get $m_{bol} = 1.34 - 0.25 = 1.09$ **(10%)**

3. Periodic orbit of α Centauri star is $P$ =79.24 years and its angle of semi major axis is α = 17.59". From the result 2a we find that the apparent bolometric magnitude of α Centauri A is $m_{bolA} = -0.11$, while the apparent bolometric magnitude of α Centauri B is $m_{bolB} = 1.09$. (two decimal places)

*1st iteration :*

Step 1 : Let $\mathcal{M}_A$ and $\mathcal{M}_B$ be the mass of Centauri A and Centauri B stars, respectively. Then, we firstly take $\mathcal{M}_A + \mathcal{M}_B = 2$

Step 2 : Inserting the values of $\mathcal{M}_A + \mathcal{M}_B$, $P$, and α to the third Kepler equation namely,

$$\left(\frac{\alpha}{p}\right)^3 = (\mathcal{M}_A + \mathcal{M}_B)P^2$$

One gets $p$ , that is

$$p = \left[\frac{\alpha^3}{(\mathcal{M}_A + \mathcal{M}_B)P^2}\right]^{1/3} = \left[\frac{(17.59)^3}{2(79.24)^2}\right]^{1/3} = [0.4334]^{1/3} = 0.7568''$$

Step 3 : We determine the magnitude of absolute bolometric using Pogson equation,

$$m_{bol} - M_{bol} = -5 - 5\log p \implies M_{bol} = m_{bol} + 5 + 5\log p$$

For α Centauri A:

$$M_{bolA} = m_{bolA} + 5 + 5\log p = -0.11 + 5 + 5\log(0.7568)$$
$$= -0.11 + 5 - 0.6051 = 4.2849$$

For α Centauri B :

$$M_{bolB} = m_{bolB} + 5 + 5\log p = 1.09 + 5 + 5\log(0.7568)$$
$$= 1.09 + 5 - 0.6051 = 5.4849$$

Step 4 : We determine the mass of both stars using luminosity mass relation,

$$M_{bol} = -10.2\log\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right) + 4.9 \implies \mathcal{M} = 10^{\left(\frac{4.9 - M_{bol}}{10.2}\right)}\mathcal{M}_\odot$$

For α Centauri A:

$$\mathcal{M}_A = 10^{\left(\frac{4.9 - M_{bolA}}{10.2}\right)}\mathcal{M}_\odot = 10^{\left(\frac{4.9 - 4.2849}{10.2}\right)}\mathcal{M}_\odot = 1.1490\mathcal{M}_\odot$$

For α Centauri B:

$$\mathcal{M}_B = 10^{\left(\frac{4.9 - M_{bolB}}{10.2}\right)}\mathcal{M}_\odot = 10^{\left(\frac{4.9 - 5.4949}{10.2}\right)}\mathcal{M}_\odot = 0.8763\mathcal{M}_\odot$$

**(30%)**

*2nd iteration* Repeating the step 1 until 4 and using the previous resulting mass in the first iteration:

Step 1 : $\mathcal{M}_A + \mathcal{M}_B = 1.1490 + 0.8763 = 2.0253$

Step 2 : $$p = \left[\frac{\alpha^3}{(\mathcal{M}_A + \mathcal{M}_B)P^2}\right]^{1/3} = \left[\frac{(17.59)^3}{(2.0253)(79.24)^2}\right]^{1/3} = 0.7536$$

Step 3 : For α Centauri A:

$$M_{bolA} = m_{bolA} + 5 + 5\log p = -0.11 + 5 + 5\log(0.7536) = 4.2757$$

For α Centauri B :

$$M_{bolB} = m_{bolB} + 5 + 5\log p = 1.09 + 5 + 5\log(0.7536) = 5.4757$$

Step 4 : For α Centauri A:

$$\mathcal{M}_A = 10^{\left(\frac{4.9 - M_{bolA}}{10.2}\right)}\mathcal{M}_\odot = 10^{\left(\frac{4.9 - 4.2757}{10.2}\right)}\mathcal{M}_\odot = 1.1513\mathcal{M}_\odot$$

For α Centauri B:

$$\mathcal{M}_B = 10^{\left(\frac{4.9 - M_{bolB}}{10.2}\right)}\mathcal{M}_\odot = 10^{\left(\frac{4.9 - 5.4757}{10.2}\right)}\mathcal{M}_\odot = 0.8781\mathcal{M}_\odot$$ **(10%)**

*3 rd Iteration* Repeating the step 1 until 4 and using the previous resulting mass in the second iteration:

Step 1 : $\mathcal{M}_A + \mathcal{M}_B = 1.1513 + 0.8781 = 2.0294$

Step 2 : $$p = \left[\frac{\alpha^3}{(\mathcal{M}_A + \mathcal{M}_B)P^2}\right]^{1/3} = \left[\frac{(17.59)^3}{(2.0294)(79.24)^2}\right]^{1/3} = 0.75314$$

Step 3 : For α Centauri A:

$$M_{bolA} = m_{bolA} + 5 + 5\log p = -0.11 + 5 + 5\log(0.7531) = 4.2751$$

For α Centauri B :

$$M_{bolB} = m_{bolB} + 5 + 5\log p = 1.09 + 5 + 5\log(0.7531) = 5.4751$$

Step 4 : For α Centauri A:

$$\mathcal{M}_A = 10^{\left(\frac{4.9 - M_{bolA}}{10.2}\right)}\mathcal{M}_\odot = 10^{\left(\frac{4.9 - 4.2751}{10.2}\right)}\mathcal{M}_\odot = 1.1515\mathcal{M}_\odot$$

For α Centauri B:

$$\mathcal{M}_B = 10^{\left(\frac{4.9 - M_{bolB}}{10.2}\right)}\mathcal{M}_\odot = 10^{\left(\frac{4.9 - 5.4751}{10.2}\right)}\mathcal{M}_\odot = 0.8782\mathcal{M}_\odot$$

**(0%)**

*4 th Iteration* Repeating the step 1 until 4 and using the previous resulting mass in the third iteration:

Step 1 : $\mathcal{M}_A + \mathcal{M}_B = 1.1515 + 0.8782 = 2.0297$

Step 2 : $$p = \left[\frac{\alpha^3}{(\mathcal{M}_A + \mathcal{M}_B)P^2}\right]^{1/3} = \left[\frac{(17.59)^3}{(2.0297)(79.24)^2}\right]^{1/3} = 0.7531$$

Step 3 : For α Centauri A:

$$M_{bolA} = m_{bolA} + 5 + 5\log p = -0.11 + 5 + 5\log(0.7531) = 4.2753$$

For α Centauri B :

$$M_{bolB} = m_{bolB} + 5 + 5\log p = 1.09 + 5 + 5\log(0.7531) = 5.4743$$

Step 4 : For α Centauri A:

$$\mathcal{M}_{\mathrm{A}} = 10^{\left(\frac{4.9 - M_{bolA}}{10.2}\right)} \mathcal{M}_{\odot} = 10^{\left(\frac{4.9 - 4.2753}{10.2}\right)} \mathcal{M}_{\odot} = 1.1517 \mathcal{M}_{\odot}$$

For α Centauri B:

$$\mathcal{M}_{\mathrm{B}} = 10^{\left(\frac{4.9 - M_{bolB}}{10.2}\right)} \mathcal{M}_{\odot} = 10^{\left(\frac{4.9 - 5.4743}{10.2}\right)} \mathcal{M}_{\odot} = 0.8784 \mathcal{M}_{\odot}$$ **(0%)**

*5 th Iteration* Repeating the step 1 until 4 and using the previous resulting mass in the fourth iteration:

Step 1 : $\mathcal{M}_{\mathrm{A}} + \mathcal{M}_{\mathrm{B}} = 1.1517 + 0.8784 = 2.0301$

Step 2 :

$$p = \left[\frac{\alpha^3}{\left(\mathcal{M}_A + \mathcal{M}_B\right) P^2}\right]^{1/3} = \left[\frac{(17.59)^3}{(2.0301)(79.24)^2}\right]^{1/3} = 0.7530"$$

Step 3 : For α Centauri A:

$$M_{bolA} = m_{bolA} + 5 + 5\log p = -0.11 + 5 + 5\log(0.7530) = 4.2740$$

For α Centauri B :

$$M_{bolB} = m_{bolB} + 5 + 5\log p = 1.10 + 5 + 5\log(0.7530) = 5.4740$$

Step 4 : For α Centauri A:

$$\mathcal{M}_{\mathrm{A}} = 10^{\left(\frac{4.9 - M_{bolA}}{10.2}\right)} \mathcal{M}_{\odot} = 10^{\left(\frac{4.9 - 4.2740}{10.2}\right)} \mathcal{M}_{\odot} = 1.1518 \mathcal{M}_{\odot}$$

For α Centauri B:

$$\mathcal{M}_{\mathrm{B}} = 10^{\left(\frac{4.9 - M_{bolB}}{10.2}\right)} \mathcal{M}_{\odot} = 10^{\left(\frac{4.9 - 5.4740}{10.2}\right)} \mathcal{M}_{\odot} = 0.8785 \mathcal{M}_{\odot}$$ **(0%)**

The results of the fifth iteration equal the results of the fourth iteration, so

The mass of α Centauri A : $\mathcal{M}_{\mathrm{A}} = 1.1518\ \mathcal{M}_{\odot}$

The mass of α Centauri B : $\mathcal{M}_{\mathrm{B}} = 0.8785\ \mathcal{M}_{\odot}$