III. 운석의 나이

1. N(t) = N0 exp (-λt)와 D(t) = N0 - N(t)로부터

$$D(t) = N(t)e^{\lambda t} - N(t) = N(t)(e^{\lambda t} - 1)$$
$$\frac{D(t)}{N(t)} = (e^{\lambda t} - 1) \text{ or } e^{\lambda t} = \frac{D(t)}{N(t)} + 1 \text{ , so}$$
$$\lambda t = \ln\left(\frac{D(t)}{N(t)} + 1\right) \text{ and}$$
$$t = \frac{1}{\lambda}\ln\left(\frac{D(t)}{N(t)} + 1\right)$$

(10%)

2. $N(t) = \frac{1}{2} N_0$, 로 두면:

$$t_{1/2} = \ln 2/\lambda = 48.8 \times 10^9 \text{ year} = 48.8 \text{ Gyr}$$

(5%)

3. 알다시피 $^{87}\text{Rb} \rightarrow {}^{87}\text{Sr}$이고, 비 $\frac{^{87}\text{Sr}}{^{86}\text{Sr}}$를 y(종속 변수)로, $\frac{^{87}\text{Rb}}{^{86}\text{Sr}}$를 x를 (독립 변수)로 삼으면 단순 선형 회귀식을 쓸 수 있다:

$$y = a + bx$$

처음의 스트론튬을 고려하면 해당하는 식은

$$\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right) = \left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_0 + \left(e^{\lambda t} - 1\right)\left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}}\right)$$

여기서 절편 a = $\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)$;

이고 기울기 b = $e^{\lambda t} - 1)$

(20%)

4. 먼저 비를 구한다:

Sample No	운석 종류	^{86}Sr (ppm)	^{87}Rb (ppm)	^{87}Sr (ppm)	$^{87}Rb/^{86}Sr$	$^{87}Sr/^{86}Sr$
1	A	29.6	0.3	20.7	0.0101351	0.6993243
2	B	58.7	68.5	44.7	1.1669506	0.7614991
3	B	74.2	14.4	52.9	0.1940701	0.712938
4	A	40.2	7.0	28.6	0.1741294	0.7114428
5	A	19.7	0.4	13.8	0.0203046	0.7005076
6	B	37.9	31.6	28.4	0.8337731	0.7493404
7	A	33.4	4.0	23.6	0.1197605	0.7065868
8	B	29.8	105.0	26.4	3.5234899	0.885906
9	A	9.8	0.8	6.9	0.0816327	0.7040816
10	B	18.5	44.0	15.4	2.3783784	0.8324324

그리고 자료를 A형과 B형으로 나누어 정리한다:

A형

Sample No	$^{87}Rb/^{86}Sr$ (X)	$^{87}Sr/^{86}Sr$ (Y)	XY	X^2	Y^2
1	0.0101351	0.6993243	0.0070877	0.0001027	0.4890545
4	0.1741294	0.7114428	0.1238831	0.030321	0.5061509
5	0.0203046	0.7005076	0.0142235	0.0004123	0.4907109
7	0.1197605	0.7065868	0.0846212	0.0143426	0.4992649
9	0.0816327	0.7040816	0.0574761	0.0066639	0.4957309
Sum:	**0.4059623**	**3.5219431**	**0.2872916**	**0.0518425**	**2.4809121**

Λ	1.42E-11
$\overline{X}$	0.0811925
$\overline{Y}$	0.7043886
SS_{xx}	0.0188814
SS_{yy}	9.54601E-05
SS_{xy}	0.0013364
b_A	0.0707786
a_A	0.6986419

where,

SSxx : X의 제곱합 = $$\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2$$

SSyy : Y의 제곱합 = $$\sum_{i=1}^{n} Y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} Y_i\right)^2$$

SSxy : X와 Y의 곱의 합 = $\sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i$

기울기는

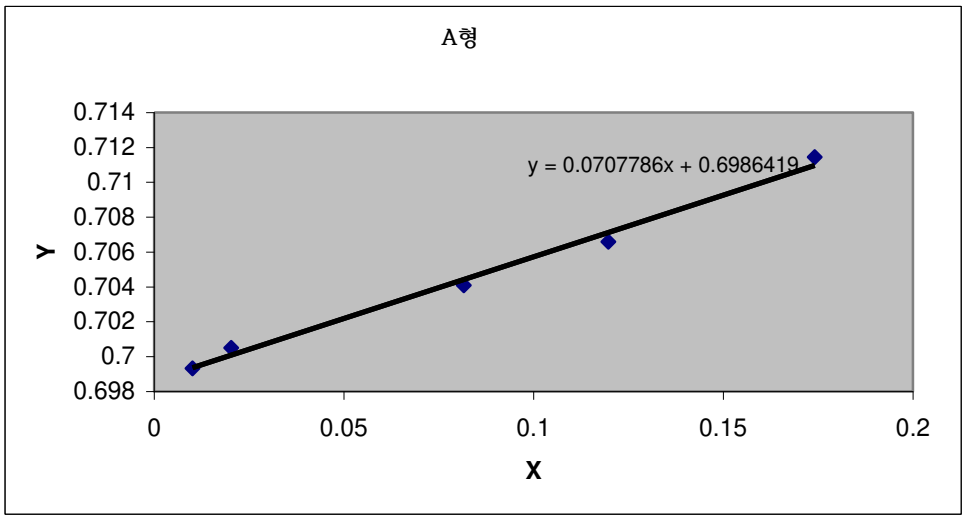
$$b_A = \left(e^{\lambda t} - 1\right) = \frac{SS_{XY}}{SS_{XX}} = \frac{\sum_{i=1}^{5} X_i Y_i - \frac{1}{5}\sum_{i=1}^{5} X_i \sum_{i=1}^{5} Y_i}{\sum_{i=1}^{5} X_i^2 - \frac{1}{5}\left(\sum_{i=1}^{5} X_i\right)^2} = \frac{0.2872916 - 0.2859552}{0.0518425 - 0.0329611} = 0.0707786 \qquad (*)$$

절편은

$$a_A = \left(\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)_0 = \bar{Y} - b\bar{X} = \overline{\left(\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)} - b\overline{\left(\frac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)}$$

$$= 0.7043886 - \ 0.0707786 \times 0.0811925 = 0.6986419$$

A형의 산점도와 선형 회귀선:



(20%)

B형

Sampel No	**$^{87}Rb/^{86}Sr$ (*X*)**	**$^{87}Sr/^{86}Sr$ (*Y*)**	***XY***	***X²***	***Y²***
2	1.1669506	0.7614991	0.8886318	1.3617737	0.5798809
3	0.1940701	0.7129380	0.1383599	0.0376632	0.5082806
6	0.8337731	0.7493404	0.6247799	0.6951776	0.561511
8	3.5234899	0.8859060	3.1214808	12.4149811	0.7848294
10	2.3783784	0.8324324	1.9798392	5.6566838	0.6929437
Sum:	**8.0966621**	**3.9421159**	**6.7530916**	**20.1662794**	**3.1274456**

λ	1.42E-11
$\bar{X}$	1.6193324
$\bar{Y}$	0.7884232
SS_{xx}	7.0550920
SS_{yy}	0.019390046
SS_{xy}	0.3694955
b_B	0.0523729
a_B	0.7036141

기울기 bB = $e^{\lambda t}-1)$

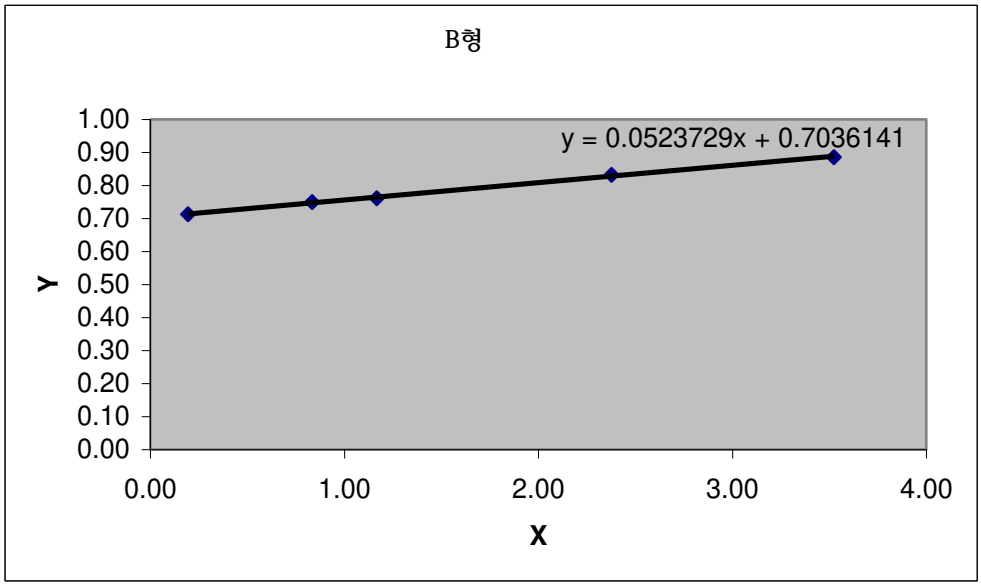
$$=\frac{SS_{XY}}{SS_{XX}}=\frac{\sum_{i=1}^{5}X_iY_i-\frac{1}{5}\sum_{i=1}^{5}X_i\sum_{i=1}^{5}Y_i}{\sum_{i=1}^{5}X_i^2-\frac{1}{5}\left(\sum_{i=1}^{5}X_i\right)^2}=\frac{6.7530916-6.3835961}{20.1662794-13.1111874}=0.0523729$$

절편은

$$a_B=\left(\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)_0=\bar{Y}-b_B\bar{X}=\overline{\left(\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)}-b_B\overline{\left(\frac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)}=0.7884232-\ 0.0523729\ \times 1.6193324$$

$$=0.7036141$$

B형의 산점도와 선형 회귀선:



(20%)

5. 운석이 두 종류이므로 나이를 따로 계산해야 하며, λ = 1.42 $\times 10^{-11}$. As $b = \left(e^{\lambda t}-1\right)$, 이면 $e^{\lambda t} = b+1$.
a와 b를 구한 뒤

$$t = \frac{1}{\lambda}\ln\ (b+1)$$

곧 4.81592×10^9 year

(*)로부터

s_b	0.0040112
s_a	0.0009133
e_t	2.819138×10^8

sigma-b :

$$s_b = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}}} = \sqrt{\frac{0.0000955 - \frac{(0.0013364)^2}{0.0188814}}{3\times0.0188814}} = 0.0040112$$

t의 오차 : $e_t = \frac{1}{\lambda}\ln(s_b+1) = \frac{\ln(0.0040112+1)}{1.42\times10^{-11}} = 2.819138\times10^8$

시그마-a (처음 값의 오차) :

$$s_a = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}}\times\sum_{i=1}^{n}X_i^2} = \sqrt{\frac{0.0000955 - \frac{(0.0013364)^2}{0.0188814}}{3\times0.0188814}\times0.0518425} = 0.0009133$$

B형

A형과 마찬가지로 B형에 대해 다음을 얻는다. 여기서 $t = 3.595\times10^9$ year.

s_b	0.0013487
s_a	0.0060566
e_t	94915372.7530205

sigma-b :

$$s_b = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}}} = \sqrt{\frac{0.0193900 - \frac{0.3694955^2}{7.0550920}}{3\times7.0550920}} = 0.0013479$$

t의 오차 : $e_t = \frac{1}{\lambda}\ln(s_b+1) = \frac{\ln(0.0013479+1)}{1.42\times10^{-11}} = 9.48586\times10^7$

시그마-a (처음 값의 오차) :

$$s_a = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}}\times\sum_{i=1}^{n}X_i^2} = \sqrt{\frac{0.0193900 - \frac{0.3694955^2}{7.0550920}}{3\times7.0550920}\times20.1662794} = 0.0060532$$

A형의 나이 = $(4.816 \pm 0.282) \times 10^9$ year = 4.816 ± 0.282 Gyr

B형의 나이 = $(3.595 \pm 0.095) \times 10^9$ year = 3.595 ± 0.435 Gyr

따라서 A형이 B형보다 훨씬 오래되었다.

(20%)

6. A형의 처음 값 $(87Sr/86Sr)0 = 0.6986419 \pm 0.0009133$

B형의 처음 값 $(87Sr/86Sr)0 = 0.7036141 \pm 0.0060532$ **(5%)**