Data analysis

Question 1.

In Figure 1, part of the constellation of Ursa Major is shown. It was taken with a digital camera with a large CCD chip (17mm × 22mm). Find the focal length, *f*, of the optical system and give the error of your results.



Figure 1. Part of the constellation of Ursa Major.

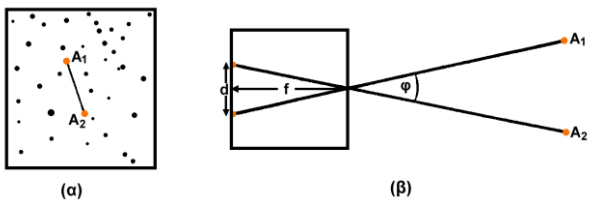


Figure 1A.Calculating the distance scale.

Answer: Calculate the angular distance, φ, (in degrees or minutes of arc), of two bright stars, whose coordinates are given in the *List of Bright stars*. Preferably these stars should be chosen to be far apart (e.g. *α UMa* [$a_1 = 11^h03^m$, $\delta_1 = +61°45'$] and *η UMa* [$a_2 = 13^h48^m$, $\delta_2 = +49°19'$]).

1. In order to calculate the angular distance of the two stars, the coordinates should be converted to decimal degrees (e.g. *α UMa* [$a_1 = 165°.75$, $\delta_1 = +61.75°$] and *η UMa* [$a_2 = 207.0°$, $\delta_2 = +49°.32$]) **(2.5 Point)**

2. Use the *cosine law* to calculate the angular distance between the two stars:

$\varphi = \arccos(\sin\delta_1 \times \sin\delta_2 + \cos\delta_1 \times \cos\delta_2 \times \cos(a_1 - a_2)) \quad \rightarrow \quad \varphi = 25°.8448$ **(10 Points)**

3. Measure the distance, d_0, of the two stars in mm in Figure 1. $d_0 = 138$ mm. **(2.5 Points**

4. The photograph in Figure 1 does not have the same dimensions as the original photograph. Measure the length, ℓ, of the photograph in mm. $\ell = 140$ mm. This length corresponds to 22 mm. Convert d_0 to the distance, *d*, of the original photograph, $d = 138 \times \frac{22}{140}$ mm. $d = 21.6857$ mm **(1.5 Points)**

4. The *image scale* $\left(\frac{\varphi}{d}\right)$ of the original photograph is $\frac{\varphi}{d} = \frac{25^o.8448}{21.6857} = 1.1918 \frac{deg}{mm}$ **(2.5 Points)**

5. The focal length is given from equation (see Figure 2): $\tan\frac{\varphi}{2}=\frac{d}{2f}$ or $f=\frac{d}{2\tan\frac{\varphi}{2}}$ from

which the focal length is calculated: $f=\frac{21.6857}{2\times 0.2294}$ or $\boldsymbol{f = 47.3}$ **mm** (**1.5 Points**)

(If errors are included **(4 Points)**

Question 2.

You are given 5 recent photographs of the solar photosphere shot at exactly the same time every two days (May 1 – May 9, 2013) in equatorial coordinates. You are also given two transparent Stonyhurst grids, which display heliocentric coordinates (heliocentric longitude, $\ell_\odot$, and heliocentric latitude, $b_\odot$). They cover the interval between April 28 to May 15. As the Earth does not orbit exactly around the Sun's equator, so, through the year, the solar equator seems to move up and down a little more than 7 degrees from the centre of the solar disc. This angle, B_0, varies sinusoidally through the year. Furthermore, the axis of rotation of the Sun, as seen from the Earth, does not coincide with the axis of rotation of the Earth. The angle on the plane of the sky between the two axes, P_0, also varies though the year. The numerical value of these angles (B_0 and P_0) are indicated on each of the 5 image of the Sun.

(1) Mark the axis of rotation of the Sun on each photograph.

(2) Choose 3 prominent sunspots that can be followed in all (or most) photographs and mark them as *S1*, *S2* and *S3* on the photos. Using the appropriate Stonyhurst grids, find their coordinates ($\ell_\odot$, $b_\odot$) for every day (May 1 to May 9) and note them down in Table 1.

Table 1

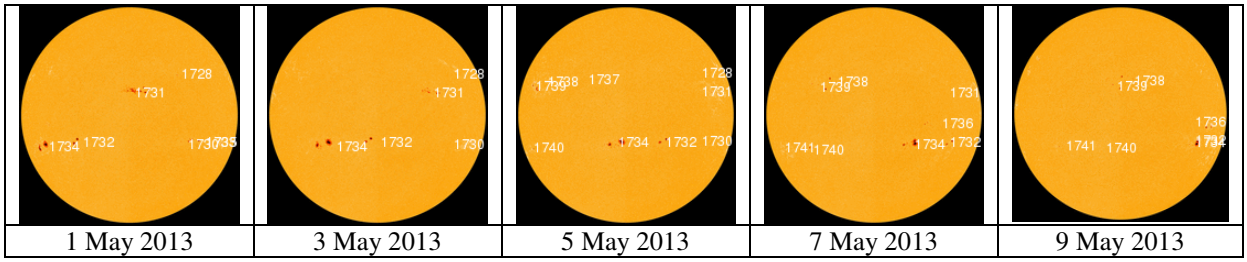
Date	Sunspot S1		Sunspot S2		Sunspot S3	
	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$
May 1						
May 3						
May 5						
May 7						
May 9						

(3) Construct the diagrams $\Delta\ell_\odot/\Delta t$ for each sunspot.

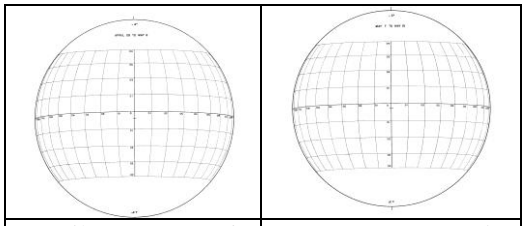
(4) Calculate its synodic period (P) of rotation **in days** for each sunspot. Write down the result for each sunspot, P_{S1}, P_{S2}, P_{S3}.

(5) Calculate the average synodic period ($P_\odot$) of rotation of the Sun in days.

Answer: Five large photographs of the solar photosphere (adapted from www.spaceweather.com) and the appropriate (and on-scale) Stonyhurst grids will be given to the students (Figure 3A)**.** The photographs given to the students, will not be annotated. Each photograph is accompanied by the angle B_0 and P_0 of the day of the observation. The student should calculate the rotation of the Sun by measuring the coordinates of at least 3 well recognized sunspots as they follow the rotation of the Sun.



1 May 2013	3 May 2013	5 May 2013	7 May 2013	9 May 2013



April 28 to May 06	May 07 to May 15

Figure 3A. Five photographs of the solar photosphere (top) and the corresponding Stonyhurst grids

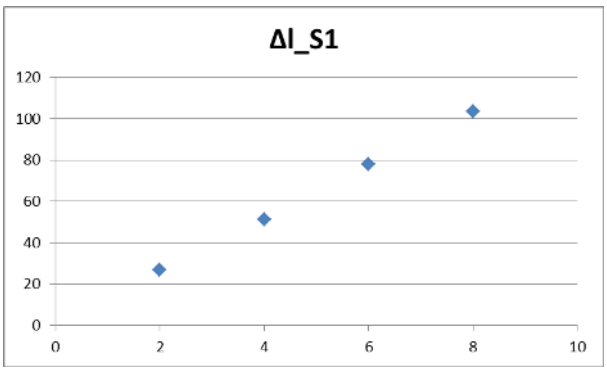
1. Correctly draw the axis of rotation of the Sun. This can be done by drawing a straight line at an angle of P_0 degrees anticlockwise from the vertical for each photograph (note: the photographs are given in equatorial coordinates.) **(4 Points)**

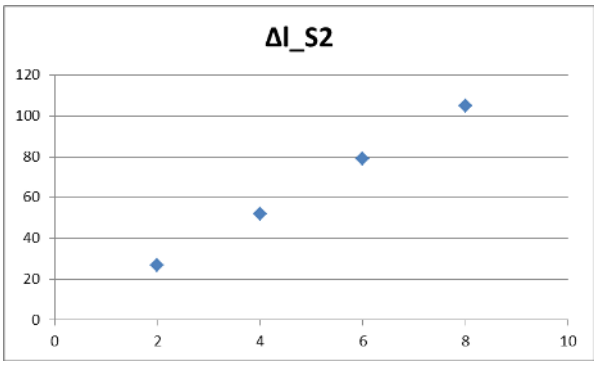
2. Choose the correct Stonyhurst grid and place it on each photograph, so that the solar axis on the grid coincides with the solar axis of the photograph. Estimate the heliocentric longitude, ($\ell_\odot$), and heliocentric latitude ($b_\odot$) with the help of the grid. Write down these coordinates in Table 1 for each of the 5 photographs.

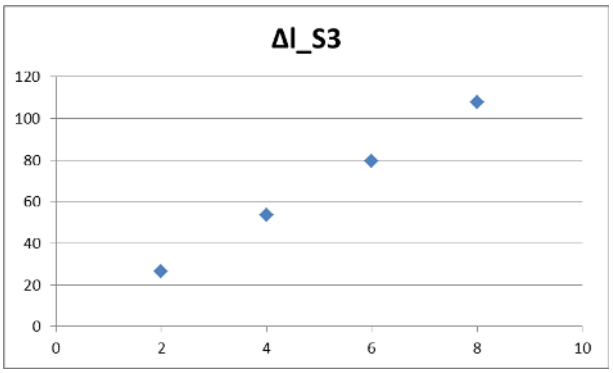
Date	Sunspot S1		Sunspot S2		Sunspot S3	
	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$
May 1	54	17.5	33	18.5	30.5	16
May 3	27	18	6	19	4	16
May 5	3	18.5	-19	18.5	-23	17
May 7	-24	18	-46	18	-49	16
May 9	-49.5	19	-72	18	-77	17

(12 Points)

3. Construct the diagram $\Delta\ell_{\odot}/\Delta t$ for each sunspot:







(2 for each graph) **(6 Points)**

4. Calculate the rotation $(\Delta\ell_{\odot}/\Delta t)$ of the Sun for each sunspot.

$$P_i = 8 * \frac{360}{\Delta\ell_{\odot i}}, \qquad P_1 = 27.8\ days, \qquad P_2 = 27.4\ days, \qquad P_3 = 26.8\ days$$

(3 for each graph) **(9 Points)**

5. Calculate the average period of the Sun

$$P_{\odot} = (P_1 + P_2 + P_3)/3 = 27.3\ days$$ **27.3 days** **(1.5 Point)**

Question 3.

Figure 2 shows a photograph of the sky in the vicinity of the Hyades open cluster. The V-filter in the Johnson's photometric system was used. Figure 3 is a chart of the region with known V-magnitudes (m_V) of several stars (note that in order to avoid confusion with the stars, no decimal point is used, i.e. a magnitude $m_V = 8.1$ is noted as "81"). Hint: some of the stars may not be in the chart.

(a) Identify as many of the *stars shown with a number and an arrow* in Figure 3 and mark them on Figure 2.

(b) Comparing the V-magnitudes of the known stars in Figure 2, estimate the V-magnitudes of the *stars shown with a number and an arrow* in Figure 3.

Answer: The human eye can easily recognize differences of the order of $\Delta m_V = 0.1 - 0.2$ mag. The student should first align the photograph with the chart, which has a different scale and orientation. Then the magnitudes of the stars can easily be recognized. Note: not all arrowed stars can be found in the photograph. (0.5 Point for each star that has been correctly identified and 0.5 point for every correct magnitude within ±0.3 magnitude **(18 Points)**

Star	3	4	5	6	7	8	9	10	11
m	7.3 – 7.9	5.3 – 5.9	5.2 – 5.8	4.6 – 5.2	5.2 – 5.8	4.5 – 5.1	4.3 – 4.9	4.9 5.5	5.3 – 5.9

Stars 1, 2 and 12 are outside the boundaries of the photograph

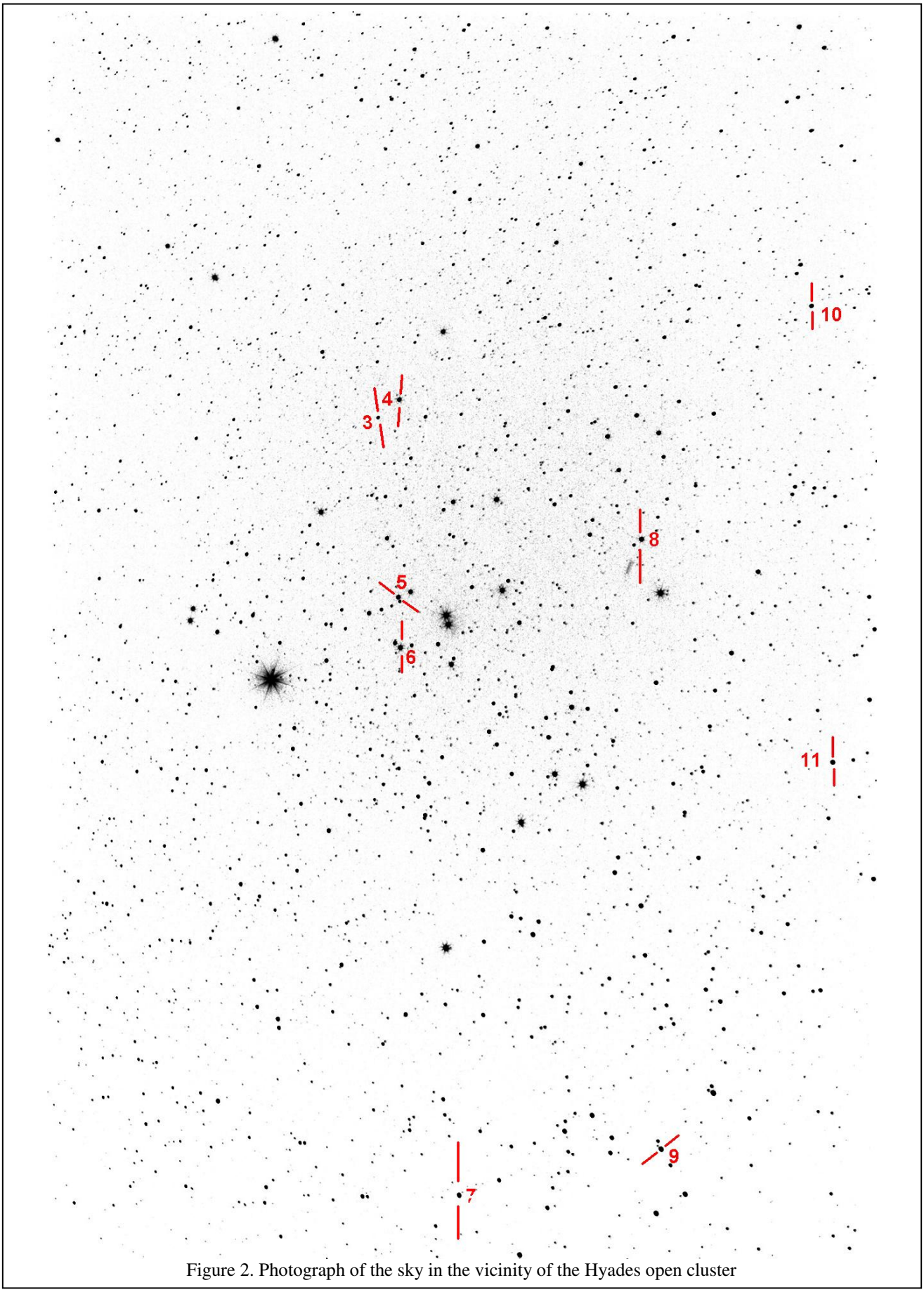


Figure 2. Photograph of the sky in the vicinity of the Hyades open cluster

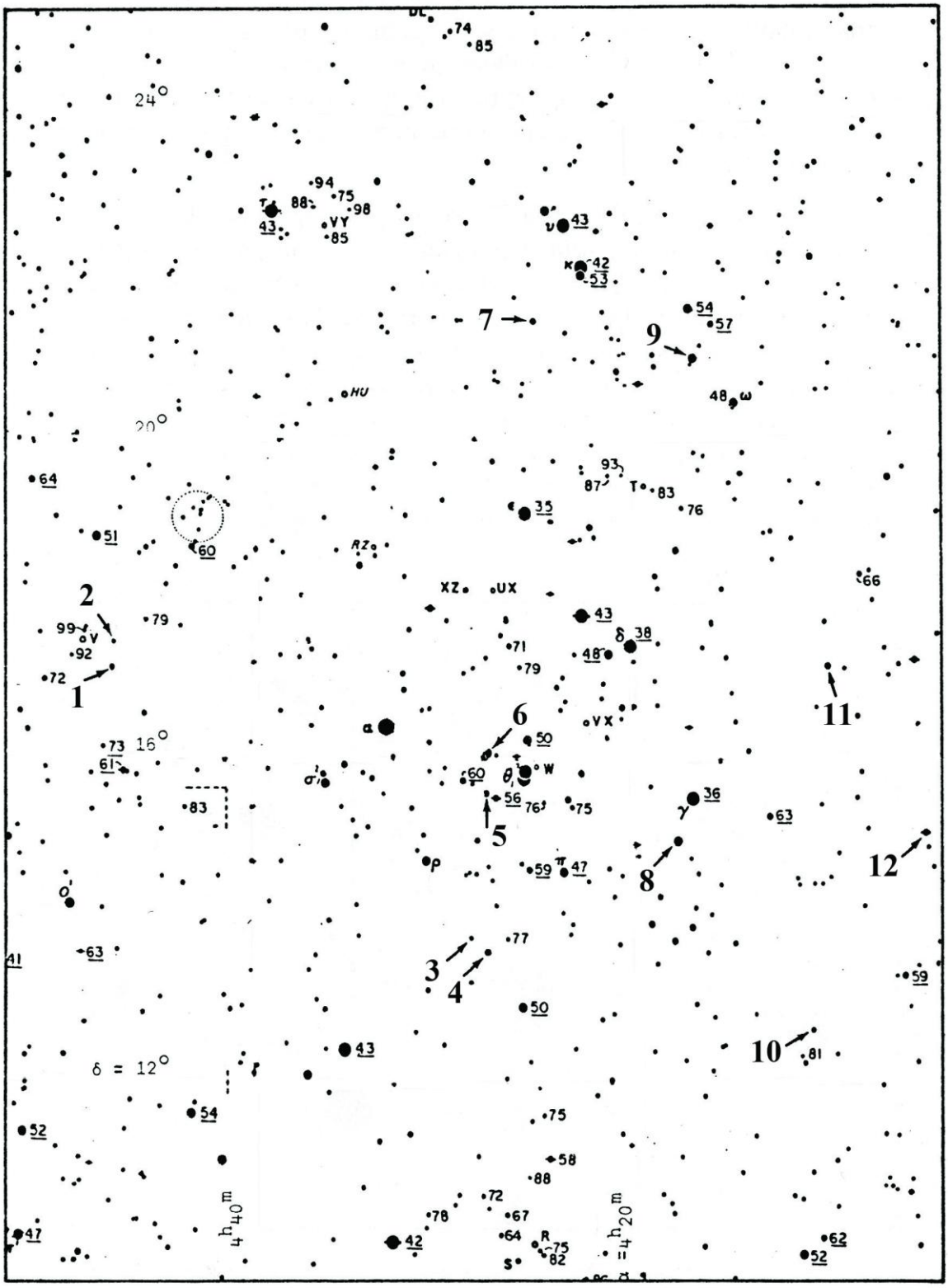


Figure 3. Sky chart of the region in the vicinity of the Hyades open cluster

Question 4.

Calculate the distance of the Hyades cluster using the *moving cluster* method (Figure 5).

1. In a *Text* file (*Hyades-stars.txt*) you are given a list of 35 stars from the field of the Hyades open cluster, observed by the Hipparcos space telescope.

The information listed in the columns of the text file for each of the 35 stars is: (a) The Hipparcos catalogue number (*HIP*). (b) Their *right ascension* (alpha – α) [h m s]. (c) Their *declination* (delta – d) [° ′ ″]. (d) Their *trigonometric parallax* (p – π) [″ × 10^3]. (e) Their *proper motion in right ascension* multiplied by cos d (mu_axcosd – $\mu_\alpha \times \cos d$) [″ × 10^3/yr]. (f) Their *proper motion in declination* (mu_d – μ_d) [″ × 10^3/yr]. (g) Their radial velocity (v_r – v_r) [km/s].

HIP	alpha	delta	p	mu_axcosd	mu_d	v_r
13834	2 58 5.08	20 40 7.7	31.41	234.79	-31.64	28.10
14838	3 11 37.67	19 43 36.1	19.44	154.61	-8.39	24.70
18170	3 53 9.96	17 19 37.8	24.14	143.97	-29.93	35.00
18735	4 0 48.69	18 11 38.6	21.99	129.49	-28.27	31.70
19554	4 11 20.20	5 31 22.9	25.89	146.86	5.00	36.60
20205	4 19 47.53	15 37 39.7	21.17	115.29	-23.86	39.28
20261	4 20 36.24	15 5 43.8	21.20	108.79	-20.67	36.20
20400	4 22 3.45	14 4 38.1	21.87	114.04	-21.40	37.80
20455	4 22 56.03	17 32 33.3	21.29	107.75	-28.84	39.65
20542	4 24 5.69	17 26 39.2	22.36	109.99	-33.47	39.20
20635	4 25 22.10	22 17 38.3	21.27	105.49	-44.14	38.60
20711	4 26 18.39	22 48 49.3	21.07	108.66	-45.83	35.60
20713	4 26 20.67	15 37 6.0	20.86	114.66	-33.30	40.80
20842	4 28 0.72	21 37 12.0	20.85	98.82	-40.59	37.50
20885	4 28 34.43	15 57 44.0	20.66	104.76	-15.01	40.17
20889	4 28 36.93	19 10 49.9	21.04	107.23	-36.77	39.37
20894	4 28 39.67	15 52 15.4	21.89	108.66	-26.39	38.90
20901	4 28 50.10	13 2 51.5	20.33	105.17	-15.08	39.90
21029	4 30 33.57	16 11 38.7	22.54	104.98	-25.14	41.00
21036	4 30 37.30	13 43 28.0	21.84	108.06	-19.71	38.80
21039	4 30 38.83	15 41 31.0	22.55	104.17	-24.29	39.56
21137	4 31 51.69	15 51 5.9	22.25	107.59	-32.38	36.00
21152	4 32 4.74	5 24 36.1	23.13	114.15	6.17	39.80
21459	4 36 29.07	23 20 27.5	22.60	109.97	-53.86	43.30
21589	4 38 9.40	12 30 39.1	21.79	101.73	-14.90	44.70
21683	4 39 16.45	15 55 4.9	20.51	82.40	-19.53	35.60
22044	4 44 25.77	11 8 46.2	20.73	98.87	-13.47	39.60
22157	4 46 1.70	11 42 20.2	12.24	67.48	-7.09	43.00
22176	4 46 16.78	18 44 5.5	10.81	73.03	-69.79	44.11
22203	4 46 30.33	15 28 19.6	19.42	91.37	-24.72	42.42
22565	4 51 22.41	18 50 23.8	17.27	79.66	-32.76	36.80
22850	4 54 58.32	19 29 7.6	14.67	63.32	-28.41	38.40
23497	5 3 5.70	21 35 24.2	20.01	68.94	-40.85	38.00
23983	5 9 19.60	9 49 46.6	18.54	63.54	-7.87	44.16
24019	5 9 45.06	28 1 50.2	18.28	55.86	-60.57	44.90

Import the *txt* file in *MS Excel*

2. Convert the coordinates in degrees (with 4 decimal points).

3. Calculate the angular distance, φ, between each of the stars and the point of convergence, which is at ($\alpha_c = 6^h7^m$, $\delta_c = +6°56'$).

4. Calculate the proper motion of each star, μ [″/yr], using $\mu_\alpha \cos\delta$ and μ_δ given in the list.

5. Use the above data to calculate the distance, r_μ, for each star using the following equation:

$$r_\mu = \frac{v_r \tan\varphi}{4.74047\,\mu}$$

where r_μ is the distance of the star in parsecs, v_r is the radial velocity of the star in km/sec, φ is the *angular distance* between the star and the point of convergence that you have already estimated in step 3, while μ is the total proper motion estimated in step 4. Do all stars belong to the Hyades cluster? You can assume that any stars whose distance from the centre of the cluster (r_μ = 46.34pc) is larger than 10 pc, are not part of the cluster.

6. Independently, calculate the distance, r_π, of each star in the list using the trigonometric parallax angle, π.

7. Find the average distance of the Hyades cluster, $\overline{r_\mu}$ and $\overline{r_\pi}$, and its standard deviation, σ_μ and σ_π, for each method (*moving cluster* and *trigonometric parallax* methods).

8. Which method is more accurate: (*i*) the *moving cluster* method, (*ii*) the *trigonometric parallax* method? Please answer with (*i*) or (*ii*).

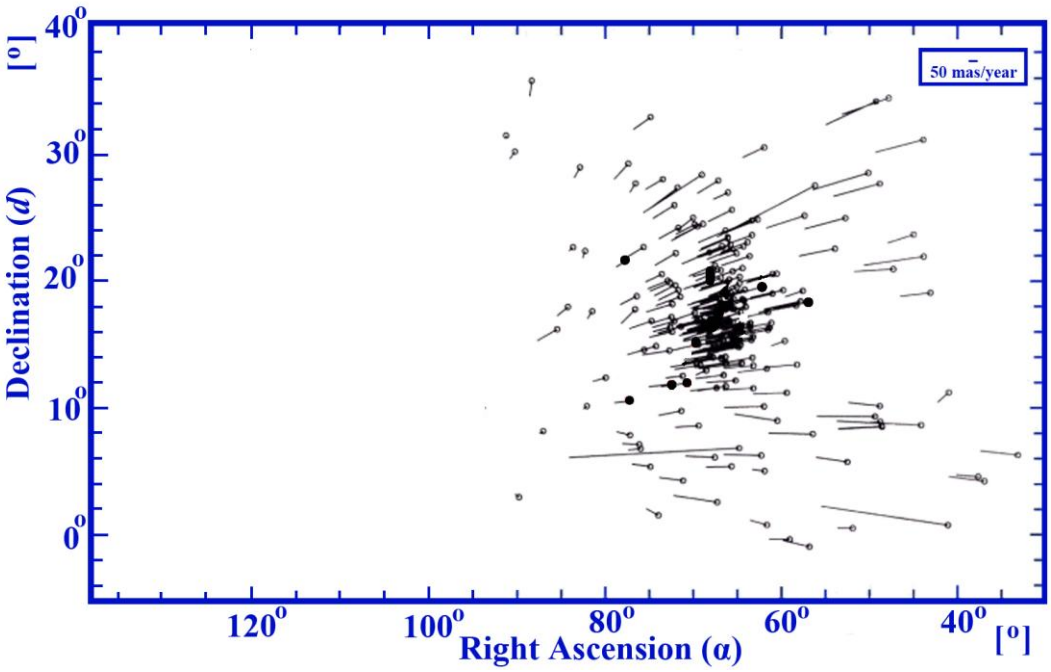


Figure 4. The distance of the Hyades open cluster using the *moving cluster* method

Answer: (1) The student should be able to import the ascii data of the *txt* file in the *MS Excel* spreadsheet application. **(12 Points)**

(2) In order to calculate the angular distance φ (next step), all coordinates should be converted into decimal degrees. The students should be able to convert the [hours, min, sec] and the [°, ′, ″] into decimal degrees, using . For the right ascension: $\alpha_{\text{deg}} = h \times 15 + \frac{m}{60} + \frac{s}{3600}$ and for the declination: $\delta_{\text{deg}} = ° + \frac{'}{60} + \frac{''}{3600}$. All coordinates are positive, so the student does not have to worry about checking the sign, which is not trivial. **(12 Points)**

(3) Using the cosine law, $\varphi = \arccos(\sin\delta_1 \times \sin\delta_2 + \cos\delta_1 \times \cos\delta_2 \times \cos(a_1 - a_2))$, the angular distance, φ, between each of the stars and the point of convergence should be calculated **(15 Points)**

(4) The student should easily calculate the proper motion of each star, by inserting the given equation, $\mu = \sqrt{(\mu_a \cos\delta)^2 + \mu_d^2}$, in the spreadsheet. **(9 Points)**

(5) Again, inserting the given equation, $r_\mu = \frac{v_r \tan\varphi}{4.74047\,\mu}$ in the spreadsheet (remembering to divide the given values of μ by 1000 to get arcseconds), the student should be able to calculate the distance, r_μ, of each star. Any star whose distance from the centre of the cluster is larger than 10 pc should be omitted from the following calculations. **(12 Points)**

(6) The trigonometric parallax distance is given by $r_\pi = \frac{1}{\pi \times 10^{-3}}$. This equation should be inserted in the spreadsheet. The result is in parsecs [pc]. **(6 Points)**

(7) Using the statistical functions *AVERAGE* and *STDEV* of MS Excel, the student should easily calculate the *average* and the *standard deviation* of r_{μ} and r_{π}. **(6 Points)**

(8) The method with the smaller standard deviation is, obviously more accurate. **(3 Point)**

(*macros* are allowed)