Problem 1 Black Hole in Milky Way

By observational facts, the scientists admit presence of a black-hole at the center of Milky Way.
At the center of Milky Way, a hypothetical black-hole (Sagittarius A*) is located. A star S* is orbiting the black-hole SA*.

In the table 1 the following data is presented: the date and the angular position coordinates $(\alpha;\beta)$ of the star S* at different moments of the observation. The coordinates represent the angular distances of the projection or the star S* in the coordinates system (U, W), centered on the SA* (see figure 1).

An angular distance of $\varphi = 1\,\text{arcsec}$ corresponds to linear distance in the plane of the sky $d = 41$ light days, therefore to a scale $S_0 = \frac{d}{\varphi} = 41\ \frac{\text{light day}}{\text{arcsec}}$.

	Date (year)	$\alpha(\text{arcsec})$	$\beta(\text{arcsec})$
1	1995.222	0.117	- 0.166
2	1997.526	0.097	- 0.189
3	1998.326	0.087	- 0.192
4	1999.041	0.077	- 0.193
5	2000.414	0.052	- 0.183
6	2001.169	0.036	- 0.167
7	2002.831	- 0.000	- 0.120
8	2003.584	- 0.016	- 0.083
9	2004.165	- 0.026	- 0.041
10	2004.585	- 0.017	0.008
11	2004.655	- 0.004	0.014
12	2004.734	0.008	0.017
13	2004.839	0.021	0.012
14	2004.936	0.037	0.009
15	2005.503	0.072	- 0.024
16	2006.041	0.088	- 0.050
17	2007.060	0.108	- 0.091

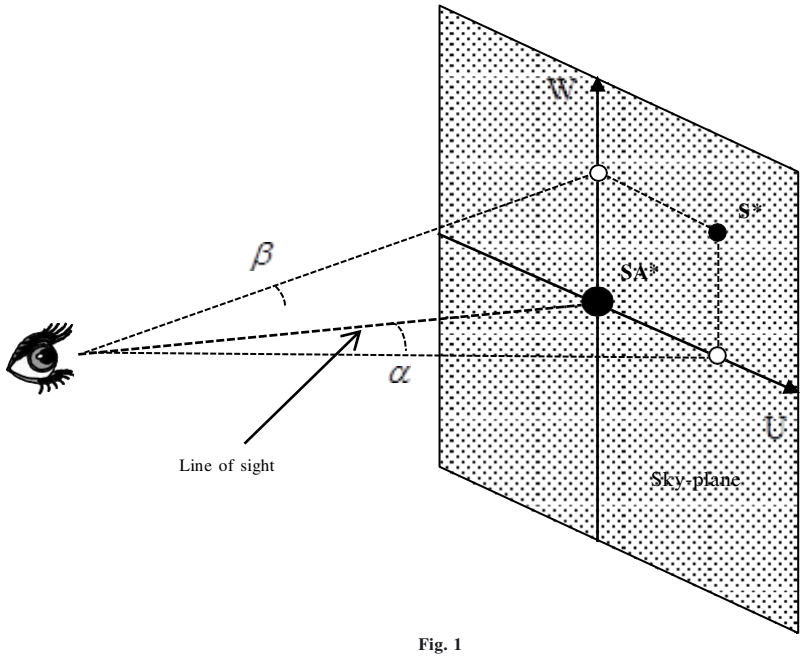


Fig. 1

By using the information provided your tasks are:

a) Plot the projection of the trajectory of the star S^* in the plane P (see figure 2). This plane is close to the observer. In this plane, $\varphi = 1\text{arcsec}$ corresponds to a linear distance $d_0 = 1200$ mm therefore the scale is $S = \frac{d_0}{\varphi} = 1200\ \frac{mm}{\text{arcsec}}$. You have to use the millimeter graph paper, carbon copy sheet of paper and the transparent sheets for an accurate plot.

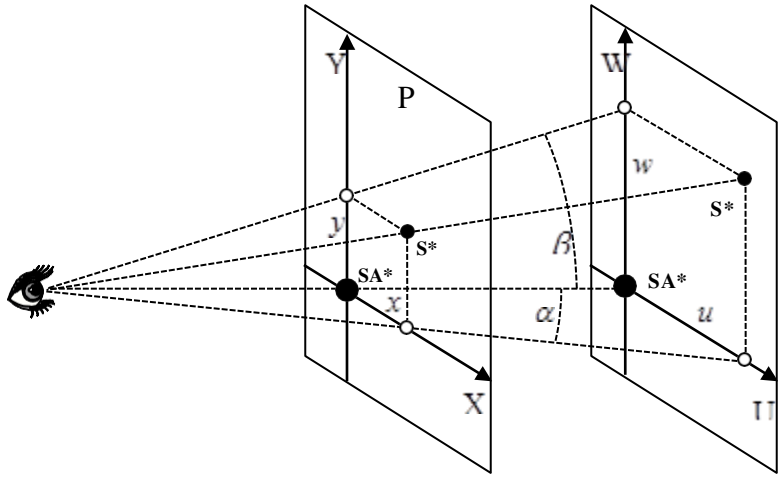


Fig. 2

b) By using the plot prove that the line of sight is normal to the actual plane of the orbit

c) Using your plot find out following elements of the real orbit of star S* around the black hole SA*:

I. a – semi-major axis (in light days units) ; b – small semi-minor axis in (in light days units); e – eccentricity;

II. r_{min} – the minimum distance between S* and SA*(in light days units); r_{max} – the maximum distance between S* and SA* (in light days units);

III. The distance from the observer to the S*;

IV. the orbiting period of star S* around SA* (obtain the best possible result by taking as many measurements as possible and by taking their arithmetic mean);

V. the total mass of the system "SA* - S*".

Presenting the intermediate and final data in tables is recommended for an accurate evaluation.

$G = 6.67 \cdot 10^{-11}$ Nm^{2}kg^{-2}