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Problem 1. Atmospheric refraction around a planet (50 points)

The light refraction in the atmosphere of a planet is an optical effect, which is due to the optical inhomogeneity of the atmosphere, thus the light propagation obey the Fermat principle. In this problem you have to find out if it is the possible that due to the atmospheric refraction an astronaut could see his back. The atmospheric absorption is negligible and the structure of the atmosphere is stable**.** Here you have some theoretical issues to help you to solve the problem:

The situation you have to investigate is sketched in the figure 1.

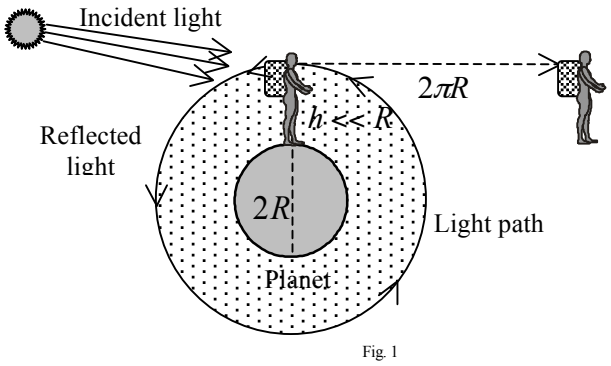
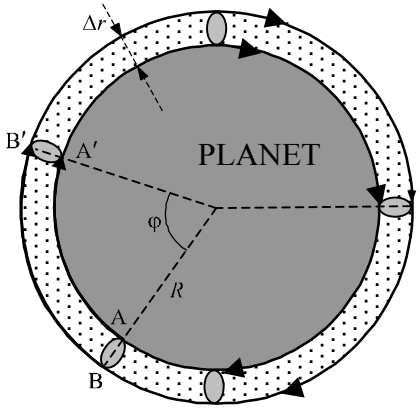


Fig. 1



The non-homogeneity of the atmosphere can be simulated by considering the atmosphere consisted of concentric spherical thin layers of thickness Δr. In one layer the refractive index is $n > 1$ due to the presence of the gas molecules. The refractive index of vacuum is $n_{vacuum} = 1$

The plane wave-front of the light AB has to remain normal on the surface of the planet and at the same distance from it, despite that the propagation occurs through an inhomogeneous atmosphere consisted of concentric spherical thin, homogenous layers.

The consequence of the model is that near the planet surface the following relationship occurred

$$n - 1 = \alpha N,$$

whith N the gas molecules concentration, and α a constant.

For Earth atmosphere near the surface:

$$n - 1 \approx 11.35 \cdot 10^{-5}.$$

In the table are given the data for 10 planets (8 from Solar System and 2 exoplanets).

R – planet radius in m; m – the mass of on gas molecule of the dominant one near the surface of the planet in atomic mass unit (1 a.m.u. $=1.66 \cdot 10^{-27}$ kg); p – atmospheric pressure near the surface of the planet in bar (1 bar = 10^5 Pa = 10^5 Nm^{-2}); T – atmosphere temperature near the surface of the planet in K; g – gravitational acceleration in ms^{-2}; You will assume that the molecules are spheres with an effective diameter d. The Boltzman-constant is $k = 1.38 \cdot 10^{-23}$ J/K.

Planet	R 10^5 m	The gas	m amu	p 10^5 Pa	T K	g m/s^2	d m
Mercur	24	CO_2	44	10^{-14}	450	3,7	$3{,}75\cdot10^{-10}$
Venus	62	CO_2	44	100	700	8,5	$3{,}75\cdot10^{-10}$
Earth	64	N_2	28	1	300	9,8	$3\cdot10^{-10}$
Mars	34	CO_2	44	10^{-2}	200	3,7	$3{,}75\cdot10^{-10}$
Jupiter	715	H_2	2	1	340	23	$2\cdot10^{-10}$
Saturn	600	H_2	2	1000	400	10,4	$2\cdot10^{-10}$
Uranus	260	H_2	2	1000	50	8,8	$2\cdot10^{-10}$
Neptun	250	H_2	2	1000	70	11,1	$2\cdot10^{-10}$
Exoplanet X	72,5	H_2	28	15	500	20	$2\cdot10^{-10}$
Exoplanet Y	25,75	CH_4	16	20	600	15	$4\cdot10^{-10}$

By using the data, you have to analyze and justify the possibility of the occurrence of the described phenomenon for each planet, considering the propagation of the light in the near the planet surface layer of the atmosphere.

a) Derive the necessary geometrical condition for producing the light refraction around a planet, if Δn is the small variance of the refractive index of the gas medium correlated with the small gas layer thickness Δr

b) Express the previous condition in terms of the thermodynamic parameters of the gas near the surface of the atmosphere i.e. the pressure, the temperature, the concentration of the gas molecules, the mass of a molecule, by calculating the value of coefficient α for the gas in the layer near the Earth surface. The refractive index n of the gas, near the surface of each planet, obey this relation $\frac{\Delta n}{n} = \frac{\alpha \Delta N}{n} \approx \alpha N$

c) Calculate the numerical values of those parameters which allow you to establish the possibility of the occurrence of the described optical phenomenon. Around which planets the phenomenon occurs. A table which centralize the data is an advantage for better understanding the solution.

Problem 1. Marking scheme Atmospheric refraction around a planet (50 points)

- a) Any relations which drive to the correct answer **10 points**
- b) **20 points**
  - thermodynamic analysis 10 points
  - the value and signification of α 10 points
- c) **20 points**
  - calculations for each planets 15 points
  - correct identification of the planets 5 puncte

a) The optical condition for the light refraction in the planet atmosphere 10 points

As seen in the figure 1the plane wave front AB has to remain perpendicular to the surface of the planet, and at the same distance from that. If we consider the thin layer near the planet which can be considered homogenous the time of light travel will be the same on the circle sectors AA′ și BB′ corresponding to the same angle φ

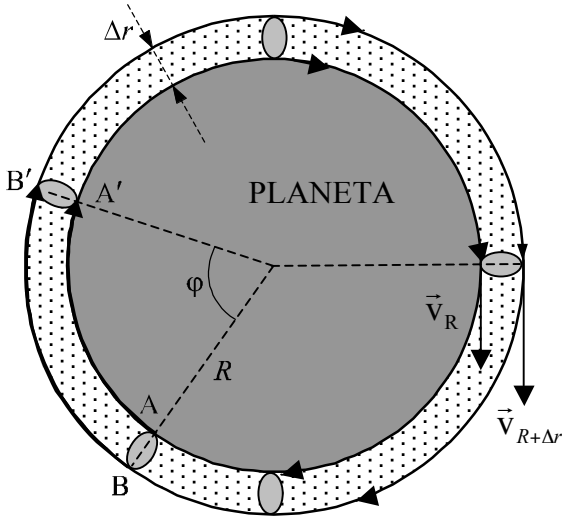


Fig.1

$$t = \frac{AA'}{v_R} = \frac{BB'}{v_{R+\Delta r}},$$

v_R - speed of light in the thin spherical homogenous layer near the planet surface – radius R

$v_{R+\Delta r}$ speed of light in the thin spherical homogenous layer near the planet surface – radius $R + \Delta r$;

$$v_{R+\Delta r} > v_R;$$

$$n_R = \frac{c}{v_R};\ n_{R+\Delta r} = \frac{c}{v_{R+\Delta r}} < n_R,$$

c speed of light in vacuum,

n_R refraction index of the atmosphere layer – radius R

$n_{R+\Delta r}$ refraction index of the atmosphere layer – radius $R + \Delta r$.

Results:

$$AA' \cdot n_R = BB' \cdot n_{R+\Delta r}; n_{R+\Delta r} < n_R;$$
$$AA' = R\varphi; BB' = (R+\Delta r)\varphi;$$
$$Rn_R = (R+\Delta r)n_{R+\Delta r};\ n_{R+\Delta r} = n_R + \Delta n,$$

where $\Delta n < 0$ is the variance of the refraction index of the atmospheric gas associated with the height Δr ;

$$Rn_R = (R+\Delta r)\cdot(n_R + \Delta n);$$
$$\Delta r \cdot \Delta n \approx 0;$$
$$-\frac{\Delta n}{n_R \Delta r} = \frac{1}{R};$$
$$n_R = n;\ -\frac{\Delta n}{n\Delta r} = \frac{1}{R};\ \frac{-\frac{\Delta n}{n}}{\Delta r} = \frac{1}{R};\ -\frac{\Delta n}{n\Delta r} = \frac{1}{R};$$

The condition of the occurrence of the refraction around the planet

$$-\frac{\Delta n / n}{\Delta r} = \frac{1}{R},$$

Using the following notations:

- theoretical curvature of the Earth surface $C_{\text{suprafata planeta}} = \frac{1}{R};$

- the curvature of the light ray in the atmosphere layer near the Earth surface $C_{light} = -\frac{\Delta n}{n\Delta r};$

Thus:

$$C_{light} = C_{\text{suprafa}\,ce\,of\,the\,\text{planet}}$$

b1) Relationship between the refractive index and the physical properties of the gas near the planet surface (pressure, temperature, molecule concentration, mass of the molecules). 10 points

According with the notations in figure 1 it can be obtain the equilibrium condition for the gas column near the Earth's surface.

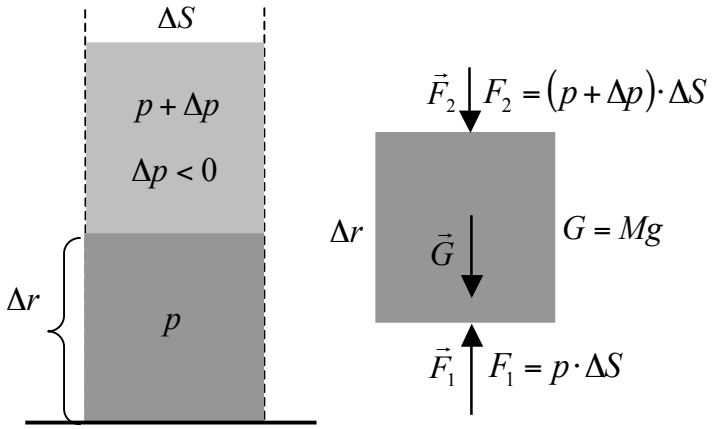


Fig. 1

$$\vec{F}_1 + \vec{G} + \vec{F}_2 = 0;$$
$$F_1 - G - F_2 = 0;$$
$$p \cdot \Delta S - Mg - (p + \Delta p) \cdot \Delta S = 0;$$
$$p \cdot \Delta S - Mg - p \cdot \Delta S - \Delta p \cdot \Delta S = 0;$$
$$-\Delta p \cdot \Delta S = Mg;$$
$$-\Delta p \cdot \Delta S = Nmg \cdot \Delta r \cdot \Delta S;$$
$$-\Delta p = Nmg \cdot \Delta r,$$

where m is the mass of a molecule, and g is the gravitational acceleration at the base of the atmosphere:

$$p = NkT; \quad \Delta p = kT\Delta N;$$
$$-kT \cdot \Delta N = Nmg \cdot \Delta r;$$

$$-\frac{\Delta N}{\Delta r} = N\frac{mg}{kT}$$ 10 points

According to the relationship

$$n - 1 = \alpha N,$$

And the fact that the $n_{\text{vid}} = 1$ the coefficient α represents the volume.

$\langle N \rangle_{\text{SI}} = \text{m}^{-3}$; $\langle \alpha \rangle_{\text{SI}} = \text{m}^3$ and $n > 1$ being proportional to the concentration of the molecules in the atmosphere.

$$n - 1 = \alpha N,$$
$$\Delta(n-1) = \Delta(\alpha N); \Delta n = \alpha \Delta N;$$
$$\frac{\Delta n}{n} = \frac{\alpha \Delta N}{n}; \; n \approx 1;$$
$$\frac{\Delta n}{n} = \frac{\alpha \Delta N}{n} \approx \alpha \Delta N,$$

Using the relation (1) :

$$\frac{\Delta n}{n} = \alpha \Delta N;\ \Delta N = \frac{\Delta n}{\alpha n};$$

$$-\frac{\Delta N}{\Delta r} = N\frac{mg}{kT}$$

$$-\frac{\frac{\Delta n}{\alpha n}}{\Delta r} = N\frac{mg}{kT};\ -\frac{\Delta n}{\alpha n \Delta r} = N\frac{mg}{kT};$$

The relationship between the light curvature and the physical properties of the atmosphere will be:

$$C_{light} = -\frac{\Delta n}{n\Delta r} = \alpha N\frac{mg}{kT} \quad (2)$$

This one can be experimental determined for each planet atmosphere and compared with the each planet surface curvature.

$$C_{light,\ \text{experimental}} = \alpha N\frac{mg}{kT};\ C_{planet} = \frac{1}{R},$$

The condition is

$$C_{\text{light}} = C_{planet};\ (3)$$

b2) The value of α, corelated with the volume of the gas molecule 10 points

α reprezents the volume of a gas molecule.

For Earth :

$$n - 1 \approx 11{,}35 \cdot 10^{-5} = \alpha N;\ p = 1\,\text{bar} = 10^5\ \text{Nm}^{-2};T = 300\ \text{K},$$

so

$$N = \frac{p}{kT} = \frac{10^5\ \text{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\text{J}}{\text{K}} \cdot 300\,\text{K}},$$

where $k = 1{,}38 \cdot 10^{-23}$ J/K (constanta lui Boltzman);

$$N \approx 2{,}41 \cdot 10^{25}\ \text{m}^{-3},$$

Is the nytrogen concentration in the Earth atmosphere;

$$11{,}35 \cdot 10^{-5} = \alpha \cdot 2{,}41 \cdot 10^{25}\ \text{m}^{-3};$$

$$\alpha \approx 4{,}70 \cdot 10^{-30}\ \text{m}^3,$$

The volume of an nytrogen molecule with the diameter $d_{N_2} \approx 3 \cdot 10^{-10}$ mis:

$$V_{1,N_2} = \frac{4\pi}{3}\left(\frac{d_{N_2}}{2}\right)^3 = \frac{4 \cdot 3{,}14}{3} \cdot \frac{9 \cdot 10^{-30}}{8}\ \text{m}^3;$$

Thus

$$V_{1,N_2} \approx 4{,}71 \cdot 10^{-30}\ \mathrm{m}^3 \approx \alpha.$$

c) Data analysis 20 points

- For Earth atmosphere considered full of nitrogen where p = 1 bar and temperature T = 300 K results:

$$N = \frac{p}{kT} = \frac{10^5\ \mathrm{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 300\,\mathrm{K}};$$

$$N \approx 2{,}41 \cdot 10^{25}\ \mathrm{m}^{-3};$$

$$V_{1,N_2} = \frac{4\pi}{3}\left(\frac{d_{N_2}}{2}\right)^3 = \frac{4 \cdot 3{,}14}{3} \cdot \frac{9 \cdot 10^{-30}}{8}\ \mathrm{m}^3;$$

$$V_{1,N_2} \approx 4{,}71 \cdot 10^{-30}\ \mathrm{m}^3 \approx \alpha;$$

$$\alpha N = 4{,}71 \cdot 10^{-30}\ \mathrm{m}^{-3} \cdot 2{,}41 \cdot 10^{25}\ \mathrm{m}^3 = 11{,}35 \cdot 10^{-5};$$

$$\alpha N \cdot \frac{mg}{kT} = 11{,}35 \cdot 10^{-5} \cdot \frac{28 \cdot 1{,}66 \cdot 10^{-27}\ \mathrm{kg} \cdot 9{,}8\ \mathrm{ms}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 3 \cdot 10^2\ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 0{,}12 \cdot 10^{-7}\ \frac{1}{\mathrm{m}}.$$

- For Venus atmosphere $T \approx 700$ K, $p \approx 100$ atm, gas CO_2:

$$N = \frac{p}{kT} = \frac{10^2 \cdot 10^5\ \mathrm{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 700\,\mathrm{K}};$$

$$N \approx 10^{27}\ \mathrm{m}^{-3}.$$

CO_2 molecule diameter

$$d_{CO_2} = 1{,}25 \cdot d_{N_2},$$

CO_2 molecule volume

$$d_{CO_2} = 3{,}75 \cdot 10^{-10}\ \mathrm{m};$$

$$V_{1,CO_2} = \frac{4\pi}{3}\left(\frac{d_{CO_2}}{2}\right)^3 = \frac{4 \cdot 3{,}14}{3} \cdot \frac{(3{,}75)^3 \cdot 10^{-30}}{8}\ \mathrm{m}^3;$$

$$V_{1,CO_2} = 27{,}59 \cdot 10^{-30}\ \mathrm{m}^3 = \alpha,$$

$$\alpha N = 27{,}59 \cdot 10^{-30}\ \mathrm{m}^{-3} \cdot 10^{27}\ \mathrm{m}^3 = 27{,}59 \cdot 10^{-3};$$

$$\alpha N \cdot \frac{mg}{kT} = 27{,}59 \cdot 10^{-3} \cdot \frac{44 \cdot 1{,}66 \cdot 10^{-27}\ \mathrm{kg} \cdot 8{,}5\ \mathrm{ms}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 7 \cdot 10^2\ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 17{,}7318 \cdot 10^{-7}\ \mathrm{m}^{-1}.$$

-For ***Mars*** :

$$N = \frac{p}{kT} = \frac{10^{-2} \cdot 10^{5}\ \mathrm{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 200\,\mathrm{K}};$$

$$N \approx 3{,}62 \cdot 10^{23}\ \mathrm{m}^{-3};$$

$$V_{1,\mathrm{CO_2}} = 27{,}59 \cdot 10^{-30}\ \mathrm{m}^{3} = \alpha;$$

$$\alpha N = 27{,}59 \cdot 10^{-30}\ \mathrm{m}^{-3} \cdot 3{,}62 \cdot 10^{23}\ \mathrm{m}^{3} = 99{,}87 \cdot 10^{-7};$$

$$\alpha N \cdot \frac{mg}{kT} = 99{,}87 \cdot 10^{-7} \cdot \frac{44 \cdot 1{,}66 \cdot 10^{-27}\ \mathrm{kg} \cdot 3{,}7\ \mathrm{ms}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 2 \cdot 10^{2}\ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 9{,}77886 \cdot 10^{-10}\ \mathrm{m}^{-1}.$$

- For Jupiter :

$$N = \frac{p}{kT} = \frac{10^{5}\ \mathrm{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 340\,\mathrm{K}};$$

$$N \approx 2{,}13 \cdot 10^{25}\ \mathrm{m}^{-3};$$

$$V_{1,\mathrm{H_2}} = 4{,}18 \cdot 10^{-30}\ \mathrm{m}^{3} = \alpha;$$

$$\alpha N = 4{,}18 \cdot 10^{-30}\ \mathrm{m}^{-3} \cdot 2{,}13 \cdot 10^{25}\ \mathrm{m}^{3} = 8{,}9 \cdot 10^{-5};$$

$$\alpha N \cdot \frac{mg}{kT} = 8{,}9 \cdot 10^{-5} \cdot \frac{2 \cdot 1{,}66 \cdot 10^{-27}\ \mathrm{kg} \cdot 23\ \mathrm{ms}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 340\,\mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 14{,}48 \cdot 10^{-10}\ \mathrm{m}^{-1}.$$

- For ***Uranus***:

$$N = \frac{p}{kT} = \frac{10^{3} \cdot 10^{5}\ \mathrm{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 50\,\mathrm{K}};$$

$$N \approx 145 \cdot 10^{27}\ \mathrm{m}^{-3};$$

$$V_{1,\mathrm{H_2}} = 4{,}18 \cdot 10^{-30}\ \mathrm{m}^{3} = \alpha;$$

$$\alpha N = 4{,}18 \cdot 10^{-30}\ \mathrm{m}^{-3} \cdot 145 \cdot 10^{27}\ \mathrm{m}^{3} = 606{,}1 \cdot 10^{-3};$$

$$\alpha N \cdot \frac{mg}{kT} = 606{,}1 \cdot 10^{-3} \cdot \frac{2 \cdot 1{,}66 \cdot 10^{-27}\ \mathrm{kg} \cdot 8{,}8\ \mathrm{ms}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 50\,\mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 2566{,}35 \cdot 10^{-8}\ \mathrm{m}^{-1}.$$

- For ***Saturn***:

$$N = \frac{p}{kT} = \frac{10^3 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 400 \ \mathrm{K}};$$

$$N \approx 18 \cdot 10^{27} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{H_2}} = 4{,}18 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 4{,}18 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 18 \cdot 10^{27} \ \mathrm{m^3} = 75{,}24 \cdot 10^{-3};$$

$$\alpha N \cdot \frac{mg}{kT} = 75{,}24 \cdot 10^{-3} \cdot \frac{2 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 10{,}4 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 400 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 470{,}63 \cdot 10^{-9} \ \mathrm{m^{-1}}.$$

- For ***Mercury***:

$$N = \frac{p}{kT} = \frac{10^{-14} \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 450 \ \mathrm{K}};$$

$$N \approx 16 \cdot 10^{10} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{H_2}} = 27{,}59 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 27{,}59 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 16 \cdot 10^{10} \ \mathrm{m^3} = 441{,}44 \cdot 10^{-20};$$

$$\alpha N \cdot \frac{mg}{kT} = 441{,}44 \cdot 10^{-20} \cdot \frac{44 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 3{,}7 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 450 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 1921 \cdot 10^{-25} \ \mathrm{m^{-1}}.$$

- For ***Neptun***:

$$N = \frac{p}{kT} = \frac{10^3 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 70 \ \mathrm{K}};$$

$$N \approx 10^{29} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{H_2}} = 4{,}18 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 4{,}18 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 10^{29} \ \mathrm{m^3} = 4{,}18 \cdot 10^{-1};$$

$$\alpha N \cdot \frac{mg}{kT} = 4{,}18 \cdot 10^{-1} \cdot \frac{2 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 11{,}1 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 70 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 15{,}95 \cdot 10^{-6} \ \mathrm{m^{-1}}.$$

- For exoplanet ***X***:

$$N = \frac{p}{kT} = \frac{15 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 500 \ \mathrm{K}};$$

$$N \approx 2{,}17 \cdot 10^{26} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{N_2}} = 4{,}71 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 4{,}71 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 2{,}17 \cdot 10^{26} \ \mathrm{m^3} = 10{,}22 \cdot 10^{-4};$$

$$\alpha N \cdot \frac{mg}{kT} = 10{,}22 \cdot 10^{-4} \cdot \frac{28 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 20 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 500 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 137{,}68 \cdot 10^{-9} \ \mathrm{m^{-1}}.$$

- For exoplanet ***Y***:

$$N = \frac{p}{kT} = \frac{20 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 600 \ \mathrm{K}};$$

$$N \approx 2{,}41 \cdot 10^{26} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{CH_4}} = \frac{4\pi}{3}\left(\frac{d_{\mathrm{CH_4}}}{2}\right)^3 = \frac{4 \cdot 3{,}14}{3} \cdot \frac{(4)^3 \cdot 10^{-30}}{8} \ \mathrm{m^3};$$

$$V_{1,\mathrm{H_2}} = 33{,}5 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 33{,}5 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 2{,}41 \cdot 10^{26} \ \mathrm{m^3} = 80{,}7 \cdot 10^{-4};$$

$$\alpha N \cdot \frac{mg}{kT} = 80{,}7 \cdot 10^{-4} \cdot \frac{16 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 15 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 600 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 3883 \cdot 10^{-10} \ \mathrm{m^{-1}}.$$

Centralized rezults.

Planeta	p bar	m uam	T K	g m/s^2	N m^{-3}	α m^3	$\alpha Nmg / kT$ m^{-1}
Pământ	1	28	300	9,8	$2{,}41 \cdot 10^{25}$	$4{,}71 \cdot 10^{-30}$	$0{,}12 \cdot 10^{-7}$
Venus	100	44	700	8,5	10^{27}	$27{,}59 \cdot 10^{-30}$	$17{,}73 \cdot 10^{-7}$
Marte	10^{-2}	44	200	3,7	$3{,}62 \cdot 10^{23}$	$27{,}59 \cdot 10^{-30}$	$9{,}7 \cdot 10^{-10}$

Jupiter	1	2	340	23	$2{,}13\cdot10^{25}$	$4{,}18\cdot10^{-30}$	$14{,}48\cdot10^{-10}$
Uranus	1000	2	50	8,8	$145\cdot10^{27}$	$4{,}18\cdot10^{-30}$	$2566{,}35\cdot10^{-8}$
Saturn	1000	2	400	10,4	$18\cdot10^{27}$	$4{,}18\cdot10^{-30}$	$470{,}63\cdot10^{-9}$
Mercur	10^{-14}	44	450	3,7	$16\cdot10^{10}$	$27{,}59\cdot10^{-30}$	$1921\cdot10^{-25}$
Neptun	1000	2	70	11,1	10^{29}	$4{,}18\cdot10^{-30}$	$15{,}95\cdot10^{-6}$
Exo X	15	28	500	20	$2{,}17\cdot10^{26}$	$4{,}71\cdot10^{-30}$	$137{,}68\cdot10^{-9}$
Exo Y	20	16	600	15	$2{,}41\cdot10^{26}$	$33{,}5\cdot10^{-30}$	$388{,}3\cdot10^{-9}$

Planet	$\alpha Nmg/kT$ m^{-1}	R m	$1/R$ m^{-1}	$C_{light,\,\exp}$? $C_{\text{surface planet}}$
Earth	$0{,}12\cdot10^{-7}$	$64\cdot10^{5}$	$1{,}56\cdot10^{-7}$	$C_{light,\,\exp} < C_{\text{surface planet}}$
Venus	$17{,}73\cdot10^{-7}$	$62\cdot10^{5}$	$1{,}61\cdot10^{-7}$	$C_{light,\,\exp} > C_{\text{surface planet}}$
Marte	$9{,}7\cdot10^{-10}$	$34\cdot10^{5}$	$2{,}94\cdot10^{-6}$	$C_{light,\,\exp} < C_{\text{surface planet}}$
Jupiter	$14{,}48\cdot10^{-10}$	$715\cdot10^{5}$	$1{,}3\cdot10^{-8}$	$C_{light,\,\exp} < C_{\text{surface planet}}$
Uranus	$2566{,}35\cdot10^{-8}$	$26\cdot10^{6}$	$3{,}8\cdot10^{-8}$	$C_{light,\,\exp} > C_{\text{surface planet}}$
Saturn	$470{,}63\cdot10^{-9}$	$60\cdot10^{6}$	$1{,}6\cdot10^{-9}$	$C_{light,\,\exp} > C_{\text{surface planet}}$
Mercur	$1921\cdot10^{-25}$	$2{,}4\cdot10^{6}$	$416\cdot10^{-9}$	$C_{light,\,\exp} > C_{\text{surface planet}}$
Neptun	$15{,}95\cdot10^{-6}$	$25\cdot10^{6}$	$4\cdot10^{-9}$	$C_{light,\,\exp} > C_{\text{surface planet}}$
Exo X	$137{,}68\cdot10^{-9}$	$72{,}5\cdot10^{5}$	$137\cdot10^{-9}$	$C_{light,\,\exp} \cong C_{\text{surface planet}}$
Exo Y	$388{,}3\cdot10^{-9}$	$25{,}75\cdot10^{5}$	$388\cdot10^{-9}$	$C_{light,\,\exp} \cong C_{\text{surface planet}}$

e) Conclusion

- For all the Solar System planets

$$C_{light,\,\exp} \neq C_{\text{surface planet}}$$

Thus the condition is not fullfiled

- For the exoplanets X and Y

$$C_{\text{raza lumina, exp}} \approx C_{\text{suprafata planeta}}$$

Thus the astronaut can see its back