Problem 3 Extra solar tests

A. Thermodynamic test

A shuttle was launched to investigate the CO_2 atmosphere of two planets P_1 and P_2. When the shuttle was near the each planet, a radio probe was launched toward each of the planet, in vertical direction (the planet radius direction). When the radio -probe movement becomes uniform, it starts sending values of the pressure of the atmosphere. In fig. 3.1 was plotted the atmospheric pressure values (conventional units) of the planet P_1 as function of the time of descending. When the probe touched the surface of the planet P_1 it sends the value of the temperature $T_0 = 700\,\text{K}$ and respectively the value of gravitational acceleration $g_0 = 10\,\text{ms}^{-2}$

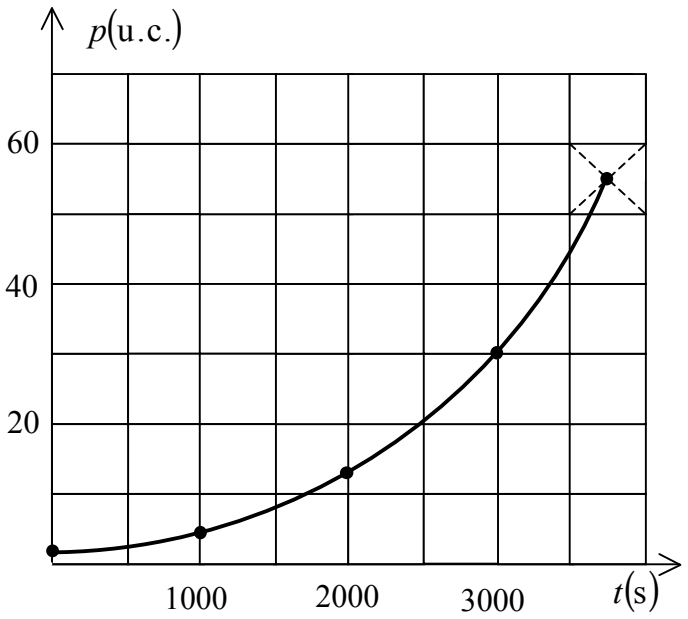


Fig. 3.1.

a) Find out the altitude h_0 from where the radio-probe R_1 starts the uniform descending and thus starts the transmitting information.

b) Find out the temperature of the planet P_1 at the altitude $h = 39{,}6\,\text{km}$. You know: The universal gas constant $R = 8{,}3\,\text{J/molK}$; the molar mass of CO_2, $\mu = 44\,\text{g/mol}$.

c) In fig. 3.2. was plotted the atmospheric pressure values (conventional units) as function of the time of descending in the planet P_2 atmosphere. When the probe touched the surface of the planet P_2, it sends the value of the temperature $T_0 = 750\,\text{K}$ and respectively the value of gravitational acceleration $g_0 = 8\,\text{ms}^{-2}$

Draw the following dependency graphs for $p = f(h)$ and $T = f(h)$ in the CO_2 atmosphere of the planet P_2.

The gravitational acceleration on each planet is constant during uniform descending of the radio –probes.

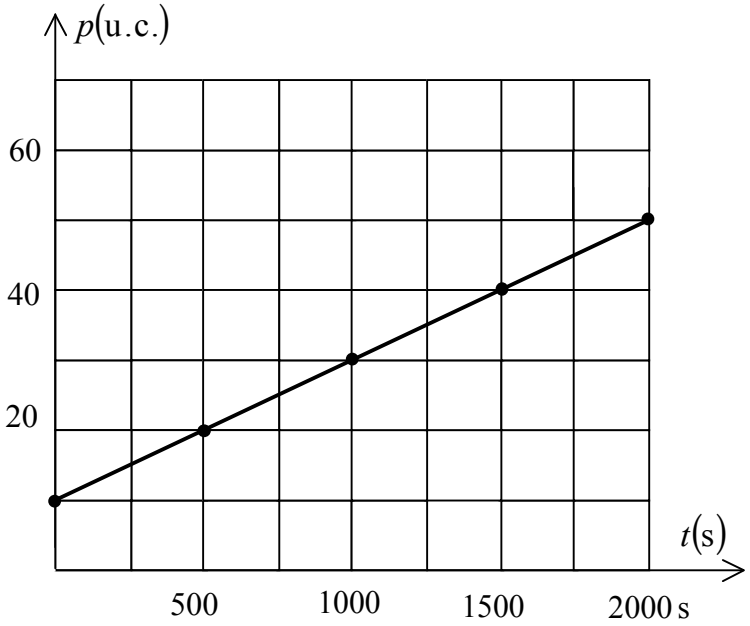


Fig. 3.2.

B. Observer on an extrasolar star

The Sirius star, located in the Canis Majoris constellation is the brightest star which can be observed on the sky with naked eye. What the observer's eye see as a single star is in matter of fact a binary star system.

The luminosity of Sirius is a consequence of two facts: its intrinsic luminosity and its closeness to Earth.

Let's assume that an observer is located on one of the planets of the Sirius system.

In the figure bellow there are represented the Sun, Sirius and the Big Dipper asterism from Ursa Major constellation.



Fig.

Determine:

a) The magnitude of Sun as seen from the observer from the planet of Sirius $m_{Sun,\text{Planet}}$

b) The magnitude of Sirius star system seen by the observer on one of the planets of Sirius. $m_{SY,\text{Planet}}$

c) The luminosity luminozitatea of „Big Dipper", $L_{\text{Big Dipper}}$;

d) the distance between Big Dipper and Earth, `

e) The geocentric angular distance between Big Dipper and Sirius, $\Delta\theta$;

f) The distance between Big Dipper and the observer from the planet of Sirius $d_{\text{Big Dipper,Planet}}$;

g) The magnitude of Big Dipper as seen by the observer from the planet of Sirius $m_{\text{Big Dipper,Planet}}$

The following data are given:

$d_{\text{Sirius},Earth} = 2{,}6\,\text{pc}$ - the Sirius – Earth distance;

$m_{\text{Sirius},Earth} = -1{,}46^{\text{m}}$ - the apparent magnitude af Sirius measured from Earth;

$L_{\text{Sirius}} = 25 \cdot L_{Sun}$ - the luminosity of Sirius;

$d_{Sun,Earth} = 1\,\text{AU}$ - distance Sun – Earth ;

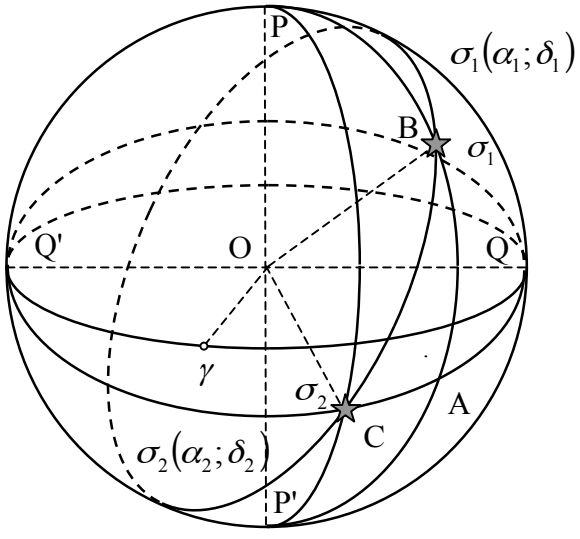
$m_{Sun,Earth} = -26{,}78^{\text{m}}$ - the apparent magnitude of Sun as seen from Earth ;

$d_{\text{Sirius,Planet}} = 10\,\text{AU}$ - the distance between Sirius and its planet where the observer is located;

$m_{\text{Big Dipper,Pamant}} \approx 2^{\text{m}}$ the apparent magnitude of the Big Dipper asterism for an observer from Earth.

In the table bellow there are information for the stars from Big Dipper asterism regardless to an observer on Earth.

Star number	Name of the star	Apparent magnitude	Distance (light year)
1	Dubhe	1,8	124
2	Merak	2,4	79
3	Phecda	2,4	84
4	Megrez	3,3	58
5	Alioth	1,8	81
6	Mizar	2,1	78
7	Alkaid	1,9	101



The equatorial of Big Dipper (σ_1) asterism and respectively of Sirius (σ_2), located on the geocentric map are :

$\alpha_{\text{Big Dipper}} = \alpha_1 = 13^{\text{h}}23^{\text{min}}55{,}5^{\text{s}}$;

$\delta_{\text{Big Dipper}} = \delta_1 = 54^{\circ}55'31''$; $\alpha_{\text{Sirius}} = \alpha_2 = 6^{\text{h}}45^{\text{min}}$;

$\delta_{\text{Sirius}} = \delta_2 = -16^{\circ}43'$.

Problem 3 A. Marking scheme Thermodynamic test 25 points

a.	5 Points
b.	10 Points
c.	10 points

The pressure and the density of the atmosphere increase with the decrease of the altitude. Th pressure at the surface of the planet is p_{max}.

In figure 1 is represented a thin atmospheric layer with thickness Δh near the surface of the planet P_1 where the atmosphere density can be considered ρ_{max}. Inside this layer the pressure and the temperature are variable, reaching the values p_{max} and T_0 near the surface.

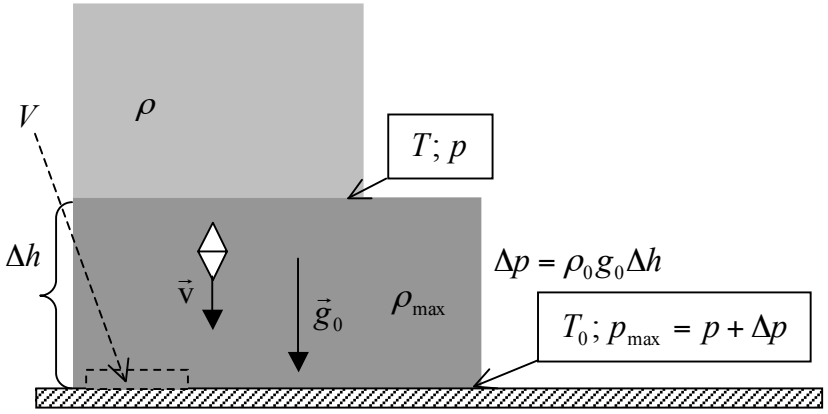


Fig. 1

The following relations can be written :

$$p_{max} = p + \Delta p;$$

$$\Delta p = \rho_{max} g_0 \Delta h,$$

Δp - the variation of the pressure due to the altitude variation Δh, g_0 the gravitational acceleration near the surface of planet P_1;

$$\Delta h = \frac{\Delta p}{\rho_{max} g_0}.$$

The superior layer where the density is constant ρ is represented too. At its base the pressure and temperature hav the values p and respectively T.

For a gas volume inside the near surface layer of the atmosphere,near the surface of the planet for v mol of de gas according with Mendeleev – Klapeyron law

$$p_{max} V = vRT_0 = \frac{m}{\mu} RT_0;$$

$$p_{\max}\mu = \frac{m}{V}RT_0 = \rho_{\max}RT_0;$$

$$\rho_{\max} = \frac{p_{\max}\mu}{RT_0}.$$

Just before touching the surface of the planet P_1 the radio- probe moves uniformly, so the distance Δh is spend with speed $\vec{v}$, in time Δt :

$$\Delta h = v\,\Delta t;$$

$$\Delta h = \frac{\Delta p}{\rho_{\max} g_0};\ \rho_{\max} = \frac{p_{\max}\mu}{RT_0};$$

$$\frac{\Delta p}{\rho_{\max} g_0} = v\,\Delta t;\ v = \frac{\Delta p}{\rho_{\max} g_0 \Delta t};\ v = \frac{\Delta p}{\frac{p_{\max}\mu}{RT_0} g_0 \Delta t};$$

$$v = \frac{RT_0 \Delta p}{g_0 \mu p_{\max} \Delta t}, (1)$$

Where accordingly to fig .2 in the point F $(\tau; p_{\max})$, the trigonometric tangent of α is:

$$\tan\alpha = \frac{\Delta p}{\Delta t}.$$

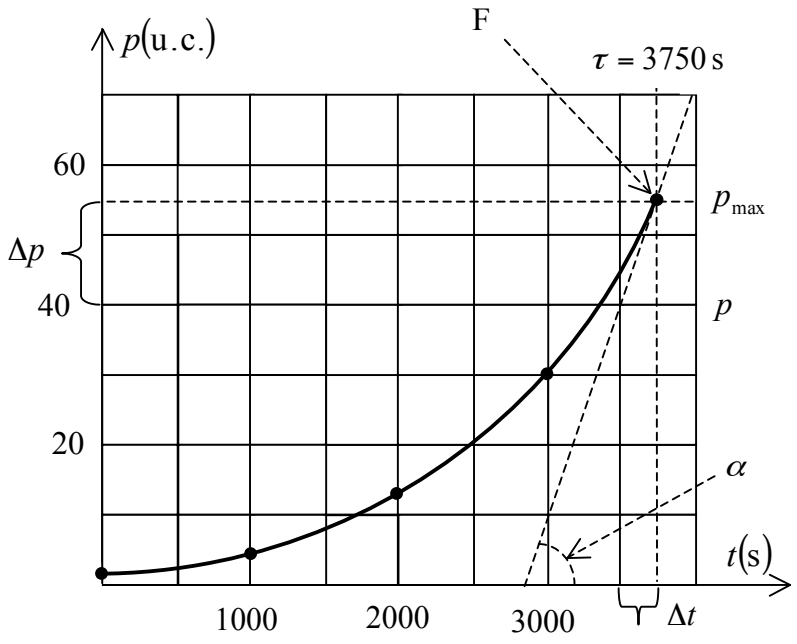


Fig. 2

The rapport :

$$\frac{\Delta p}{p_{\max}\Delta t},$$

Can be determined in the graph in the point F :

$$\mathrm{v} = \frac{\Delta p}{p_{\max}\Delta t}\frac{RT_0}{g_0\mu};$$

$\frac{\Delta p}{p_{\max}\Delta t}$	$\frac{15\,\mathrm{u.c.}}{55\,\mathrm{u.c.}\,250\,\mathrm{s}} = \frac{3}{2750}\frac{1}{\mathrm{s}}$	$\frac{45\,\mathrm{u.c.}}{55\,\mathrm{u.c.}\,750\,\mathrm{s}} = \frac{3}{2750}\frac{1}{\mathrm{s}}$

$$\mathrm{v} = \frac{3}{2750}\frac{1}{\mathrm{s}}\frac{8{,}3\,\frac{\mathrm{J}}{\mathrm{molK}}\,700\,\mathrm{K}}{10\,\frac{\mathrm{m}}{\mathrm{s}^2}\,44\,\frac{10^{-3}\ \mathrm{kg}}{\mathrm{mol}}} = 14{,}4\,\frac{\mathrm{m}}{\mathrm{s}};$$

$$h_0 = \mathrm{v}\tau;\ \tau = 3.750\ \mathrm{s};$$

$$h_0 = 54.000\,\mathrm{m} = 54\,\mathrm{km}.$$

b) Descending from the initial altitude $h_0 = 54\,\mathrm{km}$, with speed $\mathrm{v} = 14{,}4\,\frac{\mathrm{m}}{\mathrm{s}}$, the radio-probe , after 1000 s arrives at the altitude $h = 39{,}6\,\mathrm{km}$

$$t = \frac{h_0 - h}{\mathrm{v}} = \frac{14400\,\mathrm{m}}{14{,}4\,\frac{\mathrm{m}}{\mathrm{s}}} = 1000\,\mathrm{s}.$$

On Graph at altitude $h = 39{,}6\,\mathrm{km}$, the atmospheric pressure is $p_h = 4\,\mathrm{u.c.}$,- see point C on grah.

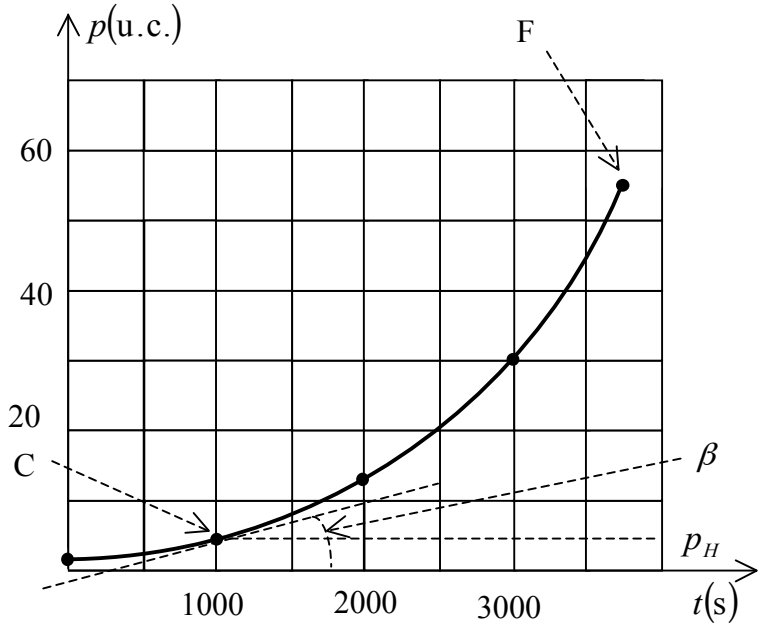


Fig. 3

In this point

$$\tan\beta = \left(\frac{\Delta p}{\Delta t}\right)_C;$$

$$\left(\frac{\Delta p}{p\Delta t}\right)_C = \frac{1}{p_h}\left(\frac{\Delta p}{\Delta t}\right)_C = \frac{1}{4\,\text{u.c.}}\frac{3\,\text{u.c.}}{500\,\text{s}} = \frac{3}{2000}\frac{1}{\text{s}};$$

$$\left(\frac{p\Delta t}{\Delta p}\right)_C = p_h\left(\frac{\Delta t}{\Delta p}\right)_C = 4\,\text{u.c.}\frac{500\,\text{s}}{3\,\text{u.c.}} = \frac{2000}{3}\text{s};$$

$$\text{v} = \left(\frac{\Delta p}{p\Delta t}\right)_C \frac{RT_h}{g_0\mu};$$

$$\text{v} = \frac{1}{p_h}\left(\frac{\Delta p}{\Delta t}\right)_C \frac{RT_h}{g_0\mu};$$

$$T_h = \frac{\text{v}\,g_0\mu}{\frac{1}{p_h}\left(\frac{\Delta p}{\Delta t}\right)_C R};\; T_h = p_h\left(\frac{\Delta t}{\Delta p}\right)_C \frac{\text{v}\,g_0\mu}{R};$$

$$T_h = \frac{2000}{3}\,\text{s}\,\frac{14{,}4\,\frac{\text{m}}{\text{s}}\,10\,\frac{\text{m}}{\text{s}^2}\,44\,\frac{10^{-3}\,\text{kg}}{\text{mol}}}{8{,}3\,\frac{\text{J}}{\text{molK}}};$$

$$T_h = 508{,}9\,\text{K}.$$

c) By using the same algorithm, corresponding to the points F, D, C, B şi A in graph on figure 4 :

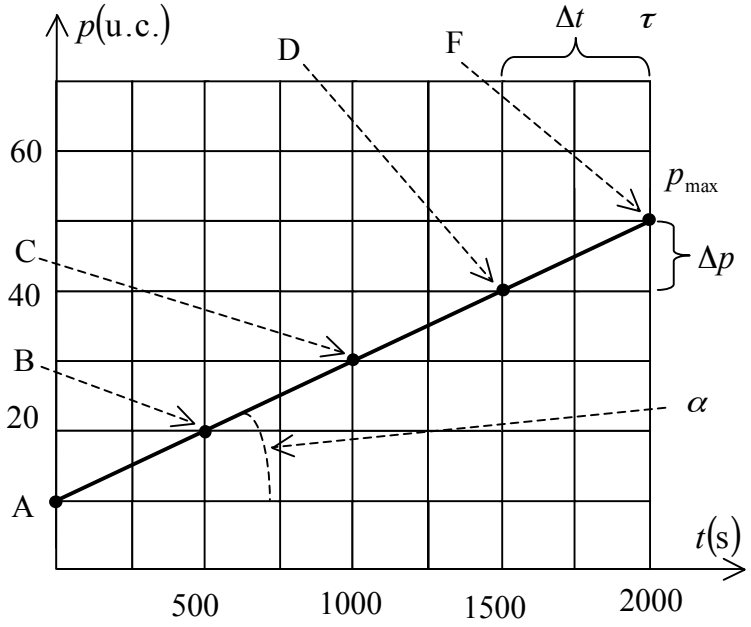


Fig. 4

$$\mathrm{v} = \frac{\Delta p}{p_{\max}\Delta t}\frac{RT_0}{g_0\mu};$$

$$\mathrm{v} = \frac{10\,\mathrm{u.c}}{50\,\mathrm{u.c.}\,500\,\mathrm{s}}\frac{8{,}3\,\frac{\mathrm{J}}{\mathrm{molK}}\,750\,\mathrm{K}}{8\,\frac{\mathrm{m}}{\mathrm{s}^2}\,44\,\frac{10^{-3}\,\mathrm{kg}}{\mathrm{mol}}} \approx 7\,\frac{\mathrm{m}}{\mathrm{s}};$$

$$h_0 = \mathrm{v}\tau = 7\,\frac{\mathrm{m}}{\mathrm{s}}\,2000\,\mathrm{s} = 14.000\,\mathrm{m} = 14\,\mathrm{km};$$

$$h_\mathrm{F} = 0;$$

$$\mathrm{F}\left(p = 50\,\mathrm{u.c.}; h = 0\,\mathrm{km}; T = 750\,\mathrm{K}\right);$$

$$T_\mathrm{D} = p_\mathrm{D}\left(\frac{\Delta t}{\Delta p}\right)_\mathrm{D}\frac{\mathrm{v}\,g_0\mu}{R} = 593{,}7\,\mathrm{K};$$

$$h_\mathrm{D} = h_0 - 7\,\frac{\mathrm{m}}{\mathrm{s}}\,1500\,\mathrm{s} = 14\,\mathrm{km} - 10.500\,\mathrm{m} = 3{,}5\,\mathrm{km};$$

$$\mathrm{D}\left(p = 40\,\mathrm{u.c.}; h = 3{,}5\,\mathrm{km}; T = 593{,}7\,\mathrm{K}\right);$$

$$T_\mathrm{C} = p_\mathrm{C}\left(\frac{\Delta t}{\Delta p}\right)_\mathrm{C}\frac{\mathrm{v}\,g_0\mu}{R} = 445{,}3\,\mathrm{K};$$

$$h_\mathrm{C} = h_0 - 7\,\frac{\mathrm{m}}{\mathrm{s}}\,1000\,\mathrm{s} = 14\,\mathrm{km} - 7.000\,\mathrm{m} = 7\,\mathrm{km};$$

$$\mathrm{C}\left(p = 30\,\mathrm{u.c.}; h = 7\,\mathrm{km}; T = 445{,}3\,\mathrm{K}\right);$$

$$T_\mathrm{B} = p_\mathrm{B}\left(\frac{\Delta t}{\Delta p}\right)_\mathrm{B}\frac{\mathrm{v}\,g_0\mu}{R} = 296{,}8\,\mathrm{K};$$

$$h_\mathrm{B} = h_0 - 7\,\frac{\mathrm{m}}{\mathrm{s}}\,500\,\mathrm{s} = 14\,\mathrm{km} - 3.500\,\mathrm{m} = 10{,}5\,\mathrm{km};$$

$$\mathrm{B}\left(p = 20\,\mathrm{u.c.}; h = 10{,}5\,\mathrm{km}; T = 296{,}8\,\mathrm{K}\right);$$

$$T_\mathrm{A} = p_\mathrm{A}\left(\frac{\Delta t}{\Delta p}\right)_\mathrm{A}\frac{\mathrm{v}\,g_0\mu}{R} = 148{,}4\,\mathrm{K};$$

$$h_\mathrm{A} = 14\,\mathrm{km};$$

$$\mathrm{A}\left(p = 10\,\mathrm{u.c.}; h = 14\,\mathrm{km}; T = 148{,}4\,\mathrm{K}\right).$$

The data are in table below which allows to plot de graph $p = f(h)$ and $T = f(h)$ for planet P_2 figure 5 and figure 6.

t	0	500 s	1000 s	1500 s	2000 s

p	10 u.c.	20 u.c.	30 u.c.	40 u.c.	50 u.c.
h	14 km	10,5 km	7 km	3,5 km	0
T	148,4 K	296,8 K	445,3 K	593,3 K	750 K

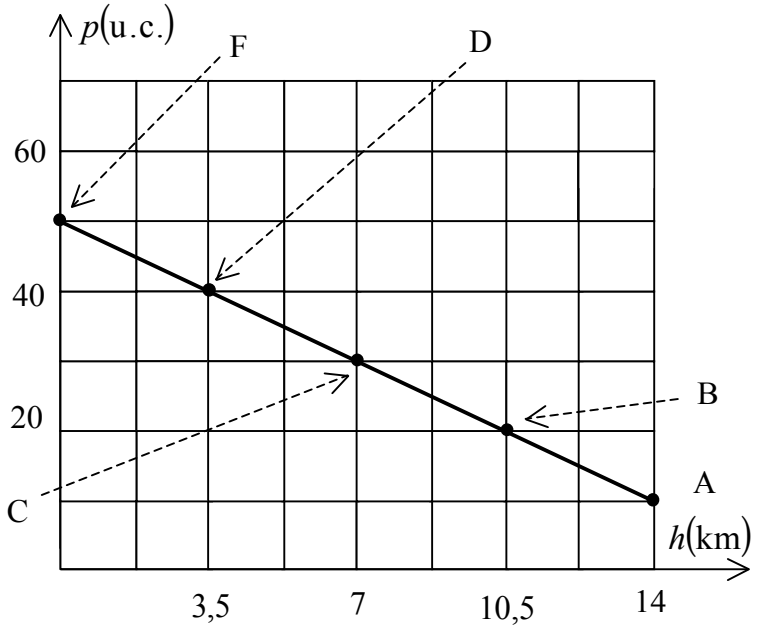


Fig. 5

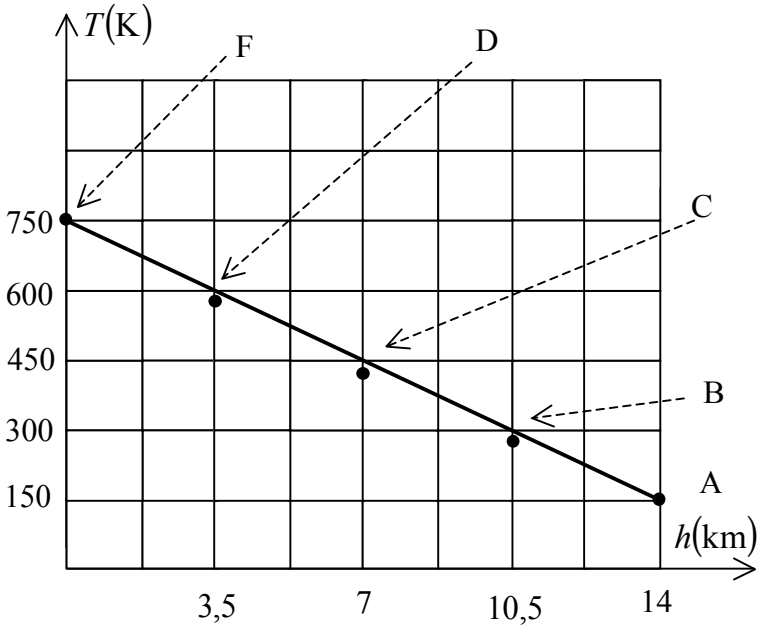


Fig. 6

Problem 3 B. Marking scheme Observer on an extrasolar star **25 points**

a.	3 Points
b.	3 Points
c.	4 Points
d.	4 Points
e.	4 Points
f.	3 Points
g.	4 Points

a) The magnitude of the **SUN as seen from the observer on the Sirrius planet**

$$\log\frac{E_{SUN,EARTH}}{E_{SUN,PLANET}} = -0{,}4\left(m_{SUN,EARTH} - m_{SUN,PLANET}\right),$$

Where: E represents the apparent brightness of the body at the corresponding distance to the observer.

$$\log\frac{\frac{L_S}{4\pi d_{S,E}^2}}{\frac{L_S}{4\pi d_{S,P}^2}} = -0{,}4\left(m_{S,E} - m_{S,P}\right),$$

$$d_{S,E} = 1\,AU;\ d_{S,P} = 2{,}6\,\text{pc} = 2{,}6\cdot 206265\,AU;$$

$$m_{S,E} = -26{,}78^{m};$$

$$m_{S,P} = 1{,}87^{m},$$

Apparent magnitude of Sun seen from the observer on one of the Sirius planet.

b) **Magnitude of Sirius, seen from the observer from its planet**:

$$\log\frac{E_{Sirius,Planeta}}{E_{Sun,Planeta}} = -0{,}4\left(m_{Sirius,Planeta} - m_{Sun,Planeta}\right),$$

$$d_{Sun,Planeta} = 2{,}6\,\text{pc} = 2{,}6\cdot 206265\,AU;$$

$$d_{Sirius,Planeta} = 10\,AU;$$

$$L_{Sirius} = 25\cdot L_{SUN};$$

$$m_{Sun,Planeta} = 1{,}87^{m};$$

$$m_{Sirius,Planeta} = -25{,}255^{m},$$

c) The luminosity of **Big Dipper**:

$$L_{Big\ Dipper} = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7,$$

where $L_1, L_2, \ldots, L_7$ are the luminosties of the seven stars of the asterism :

$$\log\frac{E_{Sun,Earth}}{E_{1,Earth}} = -0,4\left(m_{SUN,Earth} - m_{1,Earth}\right),$$

$$\log\frac{\frac{L_{Sun}}{4\pi d^2_{Sun,Earth}}}{\frac{L_1}{4\pi d^2_{1,Earth}}} = -0,4\left(m_{Sun,Earth} - m_{1,Earth}\right),$$

$$d_{1,Earth} = 124 \cdot 63240\ A\mathrm{U};$$

$$m_{Sun,Earth} = -26,78^{\mathrm{m}};\ m_{1,Earth} = 1,8^{\mathrm{m}};$$

$$L_1 \approx 225 \cdot L_{\mathrm{S}},$$

The luminosity of star 1(Dubhe) from the asterism Big Dipper.

Symilarlyfor the rest of stars in the asterism :

$$L_2 \approx 50 \cdot L_{\mathrm{S}};$$

$$L_3 \approx 60 \cdot L_{\mathrm{S}};$$

$$L_4 \approx 0,134 \cdot L_{\mathrm{S}};$$

$$L_5 \approx 52 \cdot L_{\mathrm{S}};$$

$$L_6 \approx 4 \cdot L_{\mathrm{S}};$$

$$L_7 \approx 1,37 \cdot L_{\mathrm{S}}.$$

The luminosity of the asterism Big Dipper is :

$$L_{\text{Big Dipper}} = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7;$$

$$L_{\text{Big Dipper}} = (225 + 50 + 60 + 0,134 + 52 + 4 + 1,37)L_{Sun};$$

$$L_{\text{Big Dipper}} = 392,5 \cdot L_{\mathrm{S}}un.$$

d) **The distance between Big Dipper and Earth** $d_{\text{Big Dipper},Earth}$, :

$$\log\frac{E_{\text{Big Dipper},Earth}}{E_{\text{Soare},Earth}} = -0,4\left(m_{\text{Big Dipper},Earth} - m_{\text{Soare},Earth}\right),$$

$$L_{\text{Big Dipper}} = 392,5 \cdot L_{\mathrm{S}};$$

$$d_{\text{Big Dipper},Earth} = 573 \cdot 10^3 \cdot \sqrt{392,5} \cdot d_{\text{Soare},Earth};$$

$$d_{\text{Big Dipper},Earth} \approx 11,3 \cdot 10^6\ AU.$$

e) Geocentric angular distance $\Delta\theta$, **between dintre Big Dipper and Sirius**.

The equatorial coordinates of asterism Big Dipper (σ_1) and Sirius (σ_2) located on the sphere of geocentric celestial sphere are :

$$\alpha_{\text{Big Dipper}} = \alpha_1 = 13^{\mathrm{h}}23^{\mathrm{min}}55,5^{\mathrm{s}};\ \delta_{\text{Big Dipper}} = \delta_1 = 54^{\circ}55'31'';$$

$$\alpha_{\text{Sirius}} = \alpha_2 = 6^{\text{h}}45^{\text{min}};\ \delta_{\text{Sirius}} = \delta_2 = -16^{\circ}43'.$$

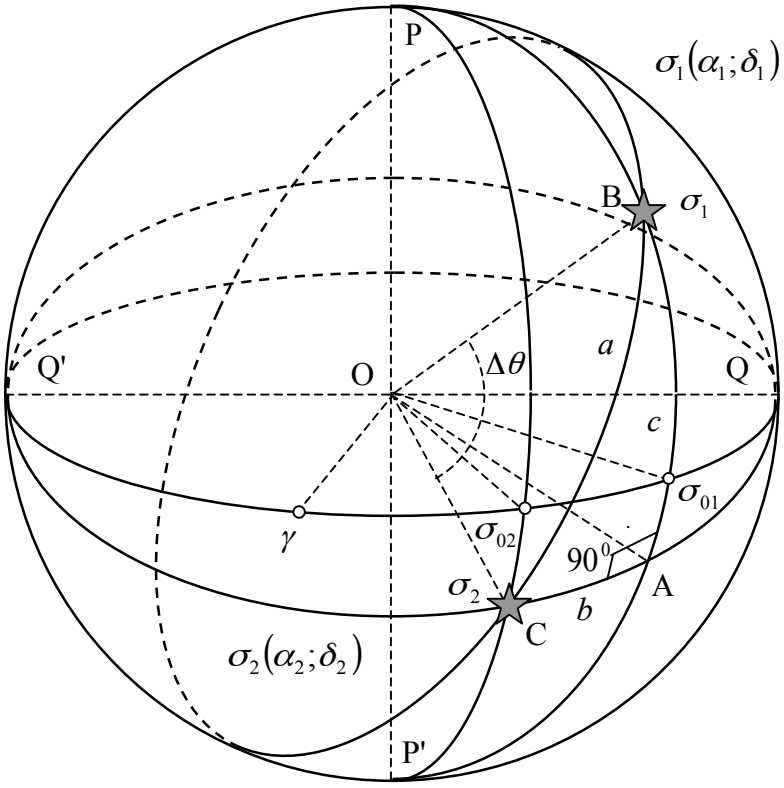


Fig.

In the spherical dreptunghic triangle ABC:

$$< (\sigma_1 \, \mathrm{O} \, \sigma_2) = \Delta\theta.$$

Also

$$< (\gamma \, \mathrm{O} \, \sigma_{01}) = \alpha_{\text{Big Dipper}} = \alpha_1;\ < (\sigma_{01} \, \mathrm{OB}) = \delta_{\text{Big Dipper}} = \delta_1;$$
$$< (\gamma \, \mathrm{O} \, \sigma_{02}) = \alpha_{\text{Sirius}} = \alpha_2;\ < (\sigma_{02} \, \mathrm{OC}) = \delta_{\text{Sirius}} = \delta_2;$$
$$< (\sigma_{01} \, \mathrm{O} \, \sigma_{02}) = \ < (\mathrm{AOC}) = \alpha_1 - \alpha_2 = \Delta\alpha;$$
$$< (\sigma_1 \, \mathrm{OA}) = \ < (\sigma_{01} \, \mathrm{O} \, \sigma_1) + < (\sigma_{01} \, \mathrm{OA}) = \delta_{\text{Big Dipper}} - \delta_{\text{Sirius}} = \Delta\delta;$$
$$< (\sigma_1 \, \mathrm{O} \, \sigma_2) = \Delta\theta.$$

Results

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A;$$
$$A = 90^0;$$
$$\cos a = \cos b \cdot \cos c;$$
$$a = \Delta\theta;\ b = \Delta\alpha;\ \ c = \Delta\delta;$$
$$\cos\Delta\theta = \cos\Delta\alpha \cdot \cos\Delta\delta;$$

$$\Delta\alpha = \alpha_1 - \alpha_2 = 13^{h}23^{min}55{,}5^{s} - 6^{h}45^{min} = 6^{h}38^{min}55{,}5' \approx 100^{\circ};$$
$$\Delta\delta = \delta_1 - \delta_2 = 54^{\circ}55'31'' + 16^{\circ}43' = 71^{\circ}38'31'' \approx 72{,}4^{\circ};$$
$$\cos\Delta\theta = \cos(100^{\circ})\cdot\cos(72{,}4^{\circ}) = (-0{,}173648177)\cdot(0{,}30236989) = -0{,}05250598;$$
$$\Delta\theta \approx 90{,}3^{\circ} \approx 90^{\circ},$$

f) The distance between **Big Dipper and the observer on sirius planet of Sirius**, $d_{\text{Big Dipper,Planeta}}$, :

$$d_{\text{Big Dipper,Planeta}} \approx d_{\text{Big Dipper,Sirius}};$$
$$d_{\text{Sirius,}Earth} \approx d_{\text{Sirius,Soare}} \approx d_{\text{Soare,Planeta}};$$
$$d_{\text{Sirius,Pamant}} = 2{,}6\ \text{pc};$$
$$d_{\text{Soare,Planeta}} = 2{,}6\ \text{pc} = 2{,}6\cdot 206265\ \text{UA} = d_{Earth};$$
$$d_{\text{Big Dipper,}Earth} \approx 11{,}3\cdot 10^{6}\ AU;$$

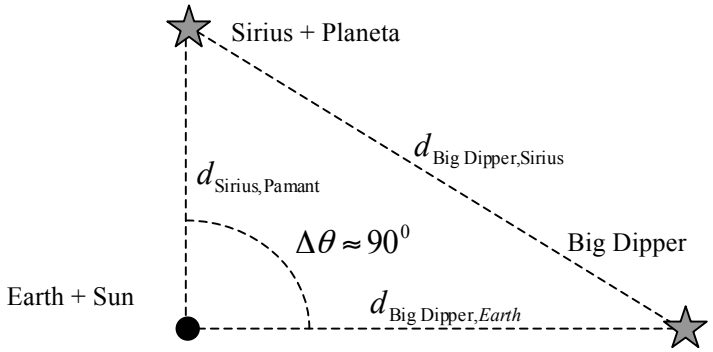


Fig.

$$d_{\text{Big Dipper,Sirius}} = \sqrt{d^{2}_{\text{Big Dipper,Pamant}} + d^{2}_{\text{Sirius,}Eaarth}} = d_{\text{Big Dipper,Planeta}} \approx 11{,}312\cdot 10^{6}\ \text{UA}.$$

g) The magnitude of **Big Dipper for an observer from the observer on planet of Sirius** :

$$\log\frac{E_{\text{Big Dipper,Planeta}}}{E_{\text{Big Dipper,}Earth}} = -0{,}4\left(m_{\text{Big Dipper,Planeta}} - m_{\text{Big Dipper,}Earth}\right),$$
$$m_{\text{Big Dipper,}Earth} \approx 2^{\text{m}};\ d_{\text{Big Dipper,}Earth} \approx 11{,}3\cdot 10^{6}\ A\text{U};$$
$$d_{\text{Big Dipper,Sirius}} = d_{\text{Big Dipper,Planeta}} \approx 11{,}312\cdot 10^{6}\ \text{A}U;$$
$$m_{\text{Big Dipper,Planeta}} = 2^{\text{m}} + 5^{\text{m}}\cdot\log\frac{11{,}312\cdot 10^{6}\ \text{AU}}{11{,}3\cdot 10^{6}\ \text{AU}};$$
$$m_{\text{Big Dipper,Planeta}} = 2^{\text{m}} + 5^{\text{m}}\cdot\log\frac{11{,}312}{11{,}3};$$
$$m_{\text{Big Dipper,Planeta}} \approx 2{,}0023^{\text{m}},$$