(D1) **Binary Pulsar**

Through systematic searches during the past decades, astronomers have found a large number of millisecond pulsars (spin period < 10 ms). Majority of these pulsars are found in binaries, with nearly circular orbits.

For a pulsar in a binary orbit, the measured pulsar spin period (P) and the measured line-of-sight acceleration (a) both vary systematically due to orbital motion. For circular orbits, this variation can be described mathematically in terms of orbital phase ϕ $(0 \leq \phi \leq 2\pi)$ as,

$$P(\phi) = P_0 + P_t \cos\phi \qquad \text{where } P_t = \frac{2\pi P_0 r}{c P_B}$$

$$a(\phi) = -a_t \sin\phi \qquad \text{where } a_t = \frac{4\pi^2 r}{P_B^2}$$

where P_B is the orbital period of the binary, P_0 is the intrinsic spin period of the pulsar and r is the radius of the orbit.

The following table gives one such set of measurements of P and a at different heliocentric epochs, T, expressed in truncated Modified Julian Days (tMJD), i.e. number of days since MJD $= 2{,}440{,}000$.

No.	T (tMJD)	P (μs)	a (m s^{-2})
1	5740.654	7587.8889	- 0.92 ± 0.08
2	5740.703	7587.8334	- 0.24 ± 0.08
3	5746.100	7588.4100	- 1.68 ± 0.04
4	5746.675	7588.5810	+ 1.67 ± 0.06
5	5981.811	7587.8836	+ 0.72 ± 0.06
6	5983.932	7587.8552	- 0.44 ± 0.08
7	6005.893	7589.1029	+ 0.52 ± 0.08
8	6040.857	7589.1350	+ 0.00 ± 0.04
9	6335.904	7589.1358	+ 0.00 ± 0.02

By plotting $a(\phi)$ as a function of $P(\phi)$, we can obtain a parametric curve. As evident from the relations above, this curve in the period-acceleration plane is an ellipse.

In this problem, we estimate the intrinsic spin period, P_0, the orbital period, P_B, and the orbital radius, r, by an analysis of this data set, assuming a circular orbit.

(D1.1) Plot the data, including error bars, in the period-acceleration plane (mark your graph as "D1.1"). **7**

(D1.2) Draw an ellipse that appears to be a best fit to the data (on the same graph "D1.1"). **2**

(D1.3) From the plot, estimate P_0, P_t and a_t, including error margins. **7**

(D1.4) Write expressions for P_B and r in terms of P_0, P_t, a_t. **4**

(D1.5) Calculate approximate value of P_B and r based on your estimations made in (D1.3), including error margins. **6**

(D1.6) Calculate orbital phase, ϕ, corresponding to the epochs of the following five observations in the above table: data rows 1, 4, 6, 8, 9. **4**

(D1.7) Refine the estimate of the orbital period, P_B, using the results in part (D1.6) in the following way:

(D1.7a) First determine the initial epoch, T_0, which corresponds to the nearest epoch of zero orbital phase before the first observation. **2**

(D1.7b) The expected time, T_{calc}, of the estimated orbital phase angle of each observation is given by, **7**

$$T_{calc} = T_0 + \left(n + \frac{\phi}{360^\circ}\right) P_B,$$

where n is the number of full cycle of orbital phases that may have elapsed between T_0 and T (or T_{calc}). Estimate n and T_{calc} for each of the five observations in part (D1.6). Note down difference $T_{\text{O}-\text{C}}$ between observed T and T_{calc}. Enter these calculations in the table given in the Summary Answersheet.

(D1.7c) Plot $T_{\text{O}-\text{C}}$ against n (mark your graph as "D1.7"). **4**

(D1.7d) Determine the refined values of the initial epoch, $T_{0,\text{r}}$, and the orbital period, $P_{\text{B,r}}$. **7**

(D2) **Distance to the Moon**

Geocentric ephemerides of the Moon for September 2015 are given in the form of a table. Each reading was taken at 00:00 UT.

Date	R.A. (α)			Dec. (δ)			Angular Size (θ)	Phase (ϕ)	Elongation
	h	m	s	°	'	"	"		Of Moon
Sep 01	0	36	46.02	3	6	16.8	1991.2	0.927	148.6° W
Sep 02	1	33	51.34	7	32	26.1	1974.0	0.852	134.7° W
Sep 03	2	30	45.03	11	25	31.1	1950.7	0.759	121.1° W
Sep 04	3	27	28.48	14	32	4.3	1923.9	0.655	107.9° W
Sep 05	4	23	52.28	16	43	18.2	1896.3	0.546	95.2° W
Sep 06	5	19	37.25	17	55	4.4	1869.8	0.438	82.8° W
Sep 07	6	14	19.23	18	7	26.6	1845.5	0.336	70.7° W
Sep 08	7	7	35.58	17	23	55.6	1824.3	0.243	59.0° W
Sep 09	7	59	11.04	15	50	33.0	1806.5	0.163	47.5° W
Sep 10	8	49	0.93	13	34	55.6	1792.0	0.097	36.2° W
Sep 11	9	37	11.42	10	45	27.7	1780.6	0.047	25.1° W
Sep 12	10	23	57.77	7	30	47.7	1772.2	0.015	14.1° W
Sep 13	11	9	41.86	3	59	28.8	1766.5	0.001	3.3° W
Sep 14	11	54	49.80	0	19	50.2	1763.7	0.005	7.8° E
Sep 15	12	39	50.01	-3	20	3.7	1763.8	0.026	18.6° E
Sep 16	13	25	11.64	-6	52	18.8	1767.0	0.065	29.5° E
Sep 17	14	11	23.13	-10	9	4.4	1773.8	0.120	40.4° E
Sep 18	14	58	50.47	-13	2	24.7	1784.6	0.189	51.4° E
Sep 19	15	47	54.94	-15	24	14.6	1799.6	0.270	62.5° E
Sep 20	16	38	50.31	-17	6	22.8	1819.1	0.363	73.9° E
Sep 21	17	31	40.04	-18	0	52.3	1843.0	0.463	85.6° E
Sep 22	18	26	15.63	-18	0	41.7	1870.6	0.567	97.6° E
Sep 23	19	22	17.51	-17	0	50.6	1900.9	0.672	110.0° E
Sep 24	20	19	19.45	-14	59	38.0	1931.9	0.772	122.8° E
Sep 25	21	16	55.43	-11	59	59.6	1961.1	0.861	136.2° E
Sep 26	22	14	46.33	-8	10	18.3	1985.5	0.933	150.0° E
Sep 27	23	12	43.63	-3	44	28.7	2002.0	0.981	164.0° E
Sep 28	0	10	48.32	0	58	58.2	2008.3	1.000	178.3° E
Sep 29	1	9	5.89	5	38	54.3	2003.6	0.988	167.4° W
Sep 30	2	7	39.02	9	54	16.1	1988.4	0.947	153.2° W

The composite graphic[1] below shows multiple snapshots of the Moon taken at different times during the total lunar eclipse, which occurred in this month. For each shot, the centre of frame was coinciding with the central north-south line of umbra.

For this problem, assume that the observer is at the centre of the Earth and angular size refers to angular diameter of the object / shadow.

[1] Credit: NASA's Scientific Visualization Studio