(D3) **Type IA Supernovae**

Supernovae of type Ia are considered very important for the measurements of large extragalactic distances. The brightening and subsequent dimming of these explosions follow a characteristic light curve, which helps in identifying these as supernovae of type Ia.

Light curves of all type Ia supernovae can be fit to the same model light curve, when they are scaled appropriately. In order to achieve this, we first have to express the light curves in the reference frame of the host galaxy by taking care of the cosmological stretching/dilation of all observed time intervals, $\Delta t_{\rm obs}$, by a factor of $(1+z)$. The time interval in the rest frame of the host galaxy is denoted by $\Delta t_{\rm gal}$.

The rest frame light curve of a supernova changes by two magnitudes compared to the peak in a time interval Δt_0 after the peak. If we further scale the time intervals by a factor of s (i.e. $\Delta t_s = s\Delta t_{\rm gal}$) such that the scaled value of Δt_0 is the same for all supernovae, the light curves turn out to have the same shape. It also turns out that s is related linearly to the absolute magnitude, $M_{\rm peak}$, at the peak luminosity for the supernova. That is, we can write

$$s = a + bM_{\rm peak}\ ,$$

where a and b are constants. Knowing the scaling factor, one can determine absolute magnitudes of supernovae at unknown distances from the above linear equation.

The table below contains data for three supernovae, including their distance moduli, μ (for the first two), their recession speed, cz, and their apparent magnitudes, $m_{\rm obs}$, at different times. The time $\Delta t_{\rm obs} \equiv t - t_{\rm peak}$ shows number of days from the date at which the respective supernova reached peak brightness. The observed magnitudes have already been corrected for interstellar as well as atmospheric extinction.

Name	SN2006TD	SN2006IS	SN2005LZ
μ (mag)	34.27	35.64	
cz ($\mathrm{km\,s^{-1}}$)	4515	9426	12060
$\Delta t_{\rm obs}$ (days)	$m_{\rm obs}$ (mag)	$m_{\rm obs}$ (mag)	$m_{\rm obs}$ (mag)
−15.00	19.41	18.35	20.18
−10.00	17.48	17.26	18.79
−5.00	16.12	16.42	17.85
0.00	15.74	16.17	17.58
5.00	16.06	16.41	17.72
10.00	16.72	16.82	18.24
15.00	17.53	17.37	18.98
20.00	18.08	17.91	19.62
25.00	18.43	18.39	20.16
30.00	18.64	18.73	20.48

(D3.1) Compute $\Delta t_{\rm gal}$ values for all three supernovae, and fill them in the given blank boxes in the data tables on the BACK side of the Summary Answersheet. On a graph paper, plot the points and draw the three light curves in the rest frame (mark your graph as "D3.1"). **15**

Solution:

Redshifts for the three supernovae are $z_1 = 0.0151$, $z_2 = 0.0314$ and $z_3 = 0.0402$. **1.5**

Filling in the three tables ($\Delta t_{\rm gal}$, third column) **3.5**

SN2006TD			
Δt_{obs}	m_{obs}	Δt_{gal}	Δt_s
(d)	(mag)	(d)	(d)
−15.00	19.41	−14.78	−20.00
−10.00	17.48	−9.85	−13.34
−5.00	16.12	−4.93	−6.67
0.00	15.74	0.00	0.00
5.00	16.06	4.93	6.67
10.00	16.72	9.85	13.34
15.00	17.53	14.78	20.00
20.00	18.08	19.70	26.67
25.00	18.43	24.63	33.34
30.00	18.64	29.56	40.01

SN2006IS			
Δt_{obs}	m_{obs}	Δt_{gal}	Δt_s
(d)	(mag)	(d)	(d)
−15.00	18.35	−14.54	−14.54
−10.00	17.26	−9.70	−9.70
−5.00	16.42	−4.85	−4.85
0.00	16.17	0.00	0.00
5.00	16.41	4.85	4.85
10.00	16.82	9.70	9.70
15.00	17.37	14.54	14.54
20.00	17.91	19.39	19.39
25.00	18.39	24.24	24.24
30.00	18.73	29.09	29.09

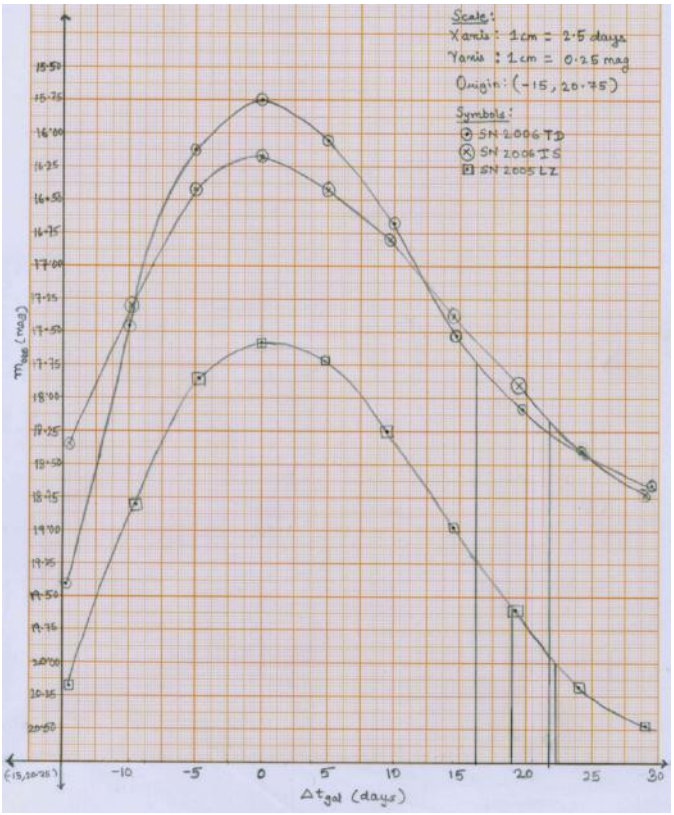
SN2005LZ			
Δt_{obs}	m_{obs}	Δt_{gal}	Δt_s
(d)	(mag)	(d)	(d)
−15.00	20.18	−14.42	−17.03
−10.00	18.79	−9.61	−11.35
−5.00	17.85	−4.81	−5.68
0.00	17.58	0.00	0.00
5.00	17.72	4.81	5.68
10.00	18.24	9.61	11.35
15.00	18.98	14.42	17.03
20.00	19.62	19.23	22.70
25.00	20.16	24.03	28.38
30.00	20.48	28.84	34.06

Full marks of 3.5 for all correct values.
Penalty for incorrect values (3×7 independent values):

Incorrect	**1-3**	**4-6**	**7-9**	**10-12**	**13-15**	**16-18**	**19-21**
Deduction	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**

The light curves in galaxy frame would appear as follows **10.0**

Graph Number: D3.1



- **Plot uses more than 50% of graph paper: 0.5**

- **Both axes labels ($\Delta t_{\rm gal}$ and $m_{\rm obs}$) present: 0.5**
- **Both dimensions of axes (days and mag) present: 0.5**
- **Ticks and values on axes (or scale written explicitly): 0.5**
- **Points correctly plotted:**
  All points correctly plotted: 5.0
  Penalty for incorrect or missing points:

Incorrect	1	2-4	5-7	8-10	11-13	14-16	17-19	20-22	23-25	26-30
Deduction	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0

- **Smooth curve through points: 1.0 per curve**

(D3.2) Take the scaling factor, s_2, for the supernova SN2006IS to be 1.00. Calculate the scaling factors, s_1 and s_3, for the other two supernovae SN2006TD and SN 2005LZ, respectively, by calculating Δt_0 for them. **5**

Solution:

From the graph D3.1, SN2006IS took 22.0 d to fade by 2 magnitudes.

That is, Δt_0(SN2006IS) = 22.0 d.

Similarly, Δt_0(SN2006TD) = 16.4 d.

And Δt_0(SN2005LZ) = 18.8 d.

Acceptable range: $\pm$1.0 days **3.0**

Thus, stretching factors for these two supernovae are

$$\boxed{s_1 = \frac{22.2}{16.4} = 1.354}$$

$$\boxed{s_3 = \frac{22.2}{18.8} = 1.181}$$ **2.0**

(D3.3) Compute the scaled time differences, $\Delta t_{\rm s}$, for all three supernovae. Write the values for $\Delta t_{\rm s}$ in the same data tables on the Summary Answersheet. On another graph paper, plot the points and draw 3 light curves to verify that they now have an identical profile (mark your graph as "D3.3"). **14**

Solution:

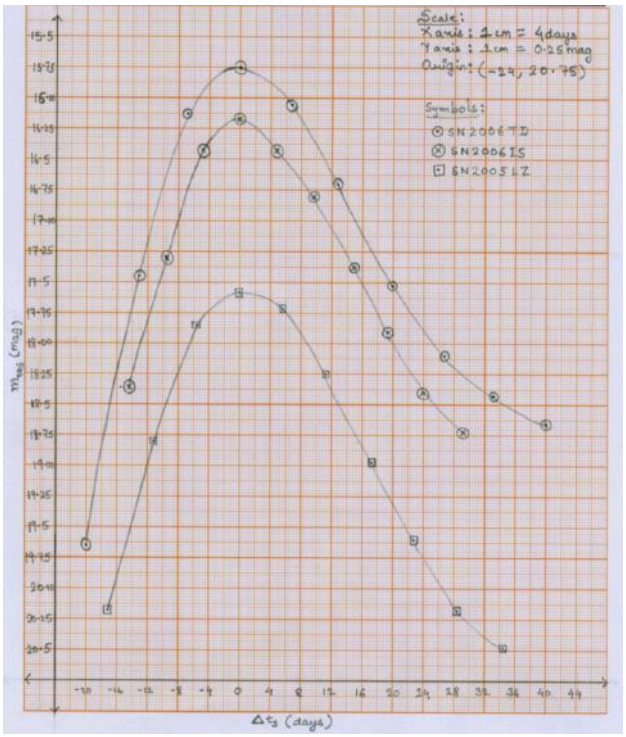
Filling the scaled values in the fourth column of the table ($\Delta t_{\rm s}$ in table above) **3.5**

Full marks of 3.5 for all correct values.
Penalty for incorrect values (3×7 independent values):

Incorrect	1-3	4-6	7-9	10-12	13-15	16-18	19-21
Deduction	0.5	1.0	1.5	2.0	2.5	3.0	3.5

The scaled light curves would appear as follows,

Graph Number: D3.3



10.5

- **Plot uses more than 50% of graph paper: 0.5**
- **Both axes labels (Δt_s and m_{obs}) present: 0.5**
- **Both dimensions of axes (days and mag) present: 0.5**
- **Ticks and values on axes (or scale written explicitly): 0.5**
- **Points correctly plotted:**
  All points correctly plotted: 5.0
  Penalty for incorrect or missing points:

Incorrect	1	2-4	5-7	8-10	11-13	14-16	17-19	20-22	23-25	26-30
Deduction	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0

- **Smooth curve through points: 1.0 per curve**
- **The curves should show identical profiles.** **0.5**

(D3.4) Calculate the absolute magnitudes at peak brightness, $M_{\text{peak},1}$, for SN2006TD and $M_{\text{peak},2}$, for SN2006IS. Use these values to calculate a and b. **6**

Solution:

To get a and b,

$$M_{\text{peak},1} = m_{\text{peak},1} - \mu_1 = 15.74 - 34.27\,\text{mag}$$
$$= -18.53\,\text{mag}$$
$$M_{\text{peak},2} = m_{\text{peak},2} - \mu_2 = 16.17 - 35.64\,\text{mag}$$
$$= -19.47\,\text{mag}$$

2.0

$$\therefore b = \frac{s_1 - s_2}{M_{\text{peak},1} - M_{\text{peak},2}} = \frac{1.354 - 1}{-18.53 - (-19.47)}\,\text{mag}^{-1} = \frac{0.354}{0.94}\,\text{mag}^{-1}$$

$\boxed{b = 0.3762\,\text{mag}^{-1}}$ **2.0**

$a = s_2 - bM_{\text{peak},2} = 1 - 0.3762 \times (-19.47) = 1 + 7.325$

$\boxed{a = 8.325}$ **2.0**

No penalty for missing mag^{-1} **in** b**.**

(D3.5) Calculate the absolute magnitude at peak brightness, $M_{\text{peak},3}$, and distance modulus, μ_3, for SN2005LZ. **4**

Solution:

$$s_3 = a + bM_{\text{peak},3}$$

$$\therefore M_{\text{peak},3} = \frac{s_3 - a}{b} = \frac{1.181 - 8.325}{0.3762}\,\text{mag} = \frac{-7.144}{0.3762}\,\text{mag}$$

$\boxed{M_{\text{peak},3} = -18.99\,\text{mag}}$ **2.0**

Distance modulus to SN2005LZ is

$\mu_3 = m_{\text{peak},3} - M_{\text{peak},3} = 17.58 - (-18.99)\,\text{mag}$

$\boxed{\mu_3 = 36.57\,\text{mag}}$ **2.0**

(D3.6) Use the distance modulus μ_3 to estimate the value of Hubble's constant, H_0. Further, estimate the characteristic age of the universe, T_{H}. **6**

Solution:

Distance to SN2005LZ is

$$d_3 = 10^{\left(\frac{\mu_3}{5}+1\right)}\text{pc} = 10^{\left(\frac{\mu_3}{5}+1-6\right)}\text{Mpc}$$

$$= 10^{\left(\frac{36.57}{5}-5\right)}\text{Mpc} = 10^{2.314}\text{Mpc}$$

$$\simeq 206\,\text{Mpc}$$

$$H_0 = \frac{cz_3}{d_3} = \frac{12060}{206}\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$$

$\boxed{H_0 = 58.5\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}}$ **4.0**

$$T_{\text{H}} = \frac{1}{H_0} = \frac{3.086 \times 10^{22}}{58.5 \times 10^3 \times 3.156 \times 10^7}\,\text{yr}$$

$\boxed{T_{\text{H}} = 16.7\,\text{Gyr}}$ **2.0**

Extra factor of $2/3$ **allowed in the value of** T_{H}**.**