(D2) **The search for dark matter** **[75 marks]**

A low surface brightness galaxy (LSB) is a diffuse galaxy with a surface brightness that, when viewed from the Earth, is at least one magnitude lower than the ambient night sky.

Some of its matter is in the form of "baryonic" matter such as neutral hydrogen gas and stars. However, most of its matter is in the form of invisible mass which is so called "dark matter". In this question, we will investigate the mass of dark matter in a galaxy, the effect of dark matter on the rotation curves of the galaxy, and the distribution of dark matter in the galaxy.

The table below provides the data of a LSB galaxy named UGC4325. The galaxy is assumed to be edge-on. At every distance r from the centre of the galaxy, we measure

1. λ_{obs}, the observed wavelength of the Hα emission line. The Hubble expansion of the Universe has already been excluded from the data.

2. V_{gas}, the contribution of the gas component to the rotation due to M_{gas}, derived from HI surface densities.

3. V_*, the contribution of the stellar component to the rotation due to M_*, derived from R-band photometry.

The rotational velocities of the test particle due to the gas component, V_{gas}, and the star component, V_*, are defined as the velocities in the plane of the galaxy that would result from the corresponding components without any external influences. These velocities are calculated from the observed baryonic mass density distributions.

r (kpc)	λ_{obs} (nm)	V_{gas} (km/s)	V_* (km/s)
0.70	656.371	2.87	20.97
1.40	656.431	6.75	32.22
2.09	656.464	14.14	40.91
2.79	656.475	20.18	46.75
3.49	656.478	24.08	50.10
4.89	656.484	28.08	47.94
6.25	656.481	29.25	45.47
7.10	656.481	27.03	47.78
9.03	656.482	25.90	45.32
12.05	656.482	21.03	42.30

The mass of dark matter $M_{\text{DM}}(r)$ within a volume of radius r can be defined in terms of the rotational velocity due to dark matter V_{DM}, the radius r and gravitational constant G,

$$M_{\mathrm{DM}}(r) = \frac{rV_{\mathrm{DM}}^2}{G}. \tag{1}$$

For the best estimate, the observed rotational velocity V_{obs} can be modeled as

$$V_{\mathrm{obs}}^2 = V_{\mathrm{gas}}^2 + V_*^2 + V_{\mathrm{DM}}^2. \tag{2}$$

The observed rotational velocity V_{obs} depends on the mass of the galaxy $M(r)$ within a volume of radius r measured from the galaxy's centre.

The mass density $\rho_{\mathrm{DM}}(r)$ of dark matter within a volume of radius r can be modeled by a galaxy density profile,

$$\rho_{\mathrm{DM}}(r) = \frac{\rho_0}{1+\left(\frac{r}{r_{\mathrm{C}}}\right)^2} \tag{3}$$

where ρ_0 and r_{C} are the central density and the core radius of the galaxy, respectively. According to the density profile, the mass of dark matter $M_{\mathrm{DM}}(r)$ within a volume of a radius r can be described by

$$M_{\mathrm{DM}}(r) = 4\pi\rho_0 r_{\mathrm{C}}^2\left[r - r_C \arctan(r/r_C)\right]. \tag{4}$$

Part 1 The mass of dark matter and rotation curves of the galaxy

(D2.1) In laboratories on Earth, Hα has an emitted wavelength λ_{emit} of 656.281 nm. Compute the observed rotational velocities of the galaxy V_{obs} and the rotational velocities due to the dark matter V_{DM} at distance r in units of km/s.

For the different values of r given in the table, compute the dynamical mass $M(r)$ and the mass of dark matter $M_{\mathrm{DM}}(r)$ in solar masses. [21]

Solution:

Redshift: $z = \dfrac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{emit}}}{\lambda_{\mathrm{emit}}}$ [0.5]

Observed rotation velocity: $V_{\mathrm{obs}} = zc$ [0.5]

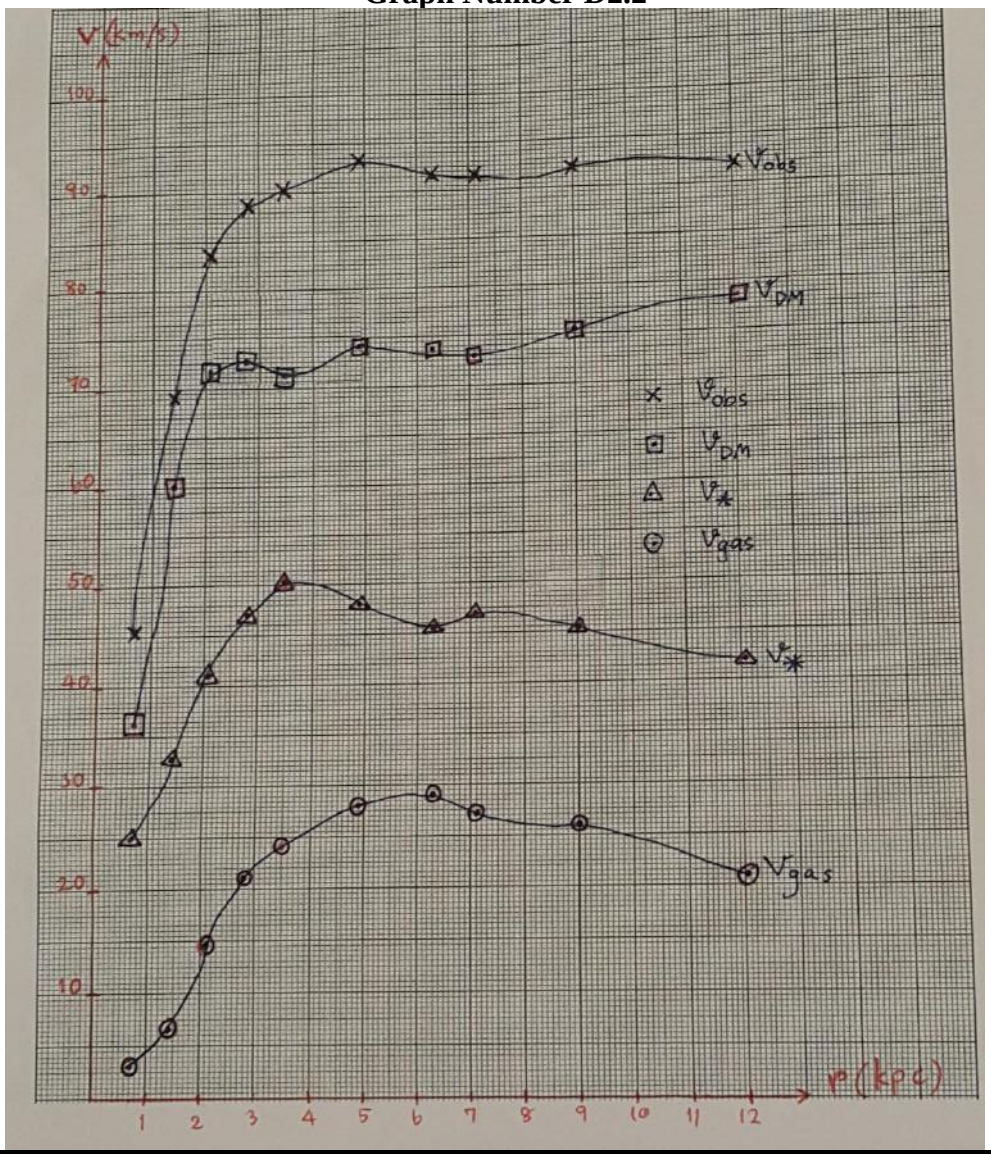
Values correctly computed: 20.0 (0.5 for each values)

r (kpc)	z	V_{obs} (km/s)	V_{DM} (km/s)	$M(r)$ $(M_\odot)$	$M_{DM}(r)(M_\odot)$
0.70	1.37×10^{-4}	41.11	35.25	2.75×10^{8}	2.02×10^{8}
1.40	2.29×10^{-4}	68.52	60.10	1.53×10^{9}	1.18×10^{9}
2.09	2.79×10^{-4}	83.60	71.52	3.40×10^{9}	2.49×10^{9}
2.79	2.96×10^{-4}	88.62	72.53	5.09×10^{9}	3.41×10^{9}
3.49	3.00×10^{-4}	89.99	70.77	6.57×10^{9}	4.06×10^{9}
4.89	3.09×10^{-4}	92.73	74.25	9.78×10^{9}	6.27×10^{9}
6.25	3.05×10^{-4}	91.36	73.65	1.21×10^{10}	7.88×10^{9}
7.10	3.05×10^{-4}	91.36	73.03	1.38×10^{10}	8.80×10^{9}
9.03	3.06×10^{-4}	91.82	75.54	1.77×10^{10}	1.20×10^{10}
12.05	3.06×10^{-4}	91.82	78.74	2.36×10^{10}	1.74×10^{10}

(D2.2) Create rotation curves of the galaxy on graph paper by plotting the points of V_{obs}, V_{DM}, V_{gas}, V_* versus the radius r and draw smooth curves through the points (mark your graph as "D2.2").

Order the contribution of the different components to the observed velocity in descending order. [16]

Graph Number D2.2



- Plot uses more than 50% of graph paper: 1.0
- Both axes labels (V and r) and their units presented: 2.0
- Ticks and values on axes (or scale written explicitly): 1.0
- All plots labels (V_{obs}, V_{DM}, V_{gas}, V_*) presented: 2.0, each label gets 0.5 mark
- Points correctly plotted: 4 (0.1 for each point)
  Values of V_{obs}, V_{DM} wrong due to wrong values of z gets a maximum of 4.0 marks
- Smooth curve through points: 4.0 (1.0 per curve)
- Order of contribution: 2.0, correct order only

(D2.3) Take a data points at small r and large r to estimate ρ_0 and r_C. Note that for large values of x, $\arctan(x) \approx \pi/2$ and at small x, $\arctan(x) \approx x - x^3/3$. [7]

Solution

Evaluating for ρ_0:

$M_{\mathrm{DM}}(r) = 4\pi\rho_0 r_C^2\left[r - r_C \arctan(r/r_C)\right]$

$M_{DM}(r) = 4\pi\rho_0 r_C^3[x - \arctan(x)]$, where $x = r/r_C$ [1.0]

$M_{DM}(r) \approx 4\pi\rho_0 r_C^3\left[x - (x - \frac{x^3}{3})\right]$, for small x [1.0]

$M_{DM}(r) \approx 4\pi\rho_0 r_C^3\left(\frac{x^3}{3}\right) = \frac{4\pi\rho_0 r^3}{3}$

$\rho_0 \approx \frac{2.02\times10^8 M_\odot \times 3}{4\pi(0.7kpc)^3} = 1.42\times10^8 M_\odot/kpc^3$ [1.5]

Selected correct data to put into the formula gets 0.5, correct answer gets 0.5 and correct unit gets 0.5.

$M_{DM}(r) = 4\pi\rho_0 r_C^2\left[r - r_C \arctan\left(\frac{r}{r_C}\right)\right]$

Evaluating for r_C (Method 1):

$M_{DM}(r) \approx 4\pi\rho_0 r_C^2\left[r - r_C\frac{\pi}{2}\right]$, [1.0]

Take the last two data points at large r, then we get (for $r \gg r_C$

$\Delta M_{DM}(r) \approx 4\pi\rho_0 r_C^2[\Delta r]$ [1.5]

$r_C = 1.01\ kpc$ [1.0]

Evaluating for r_C (Method 2):

$M_{DM}(r) \approx 4\pi\rho_0 r_C^2\left[r - r_C\frac{\pi}{2}\right]$, [1.0]

Take the last data point, we get a cubic equation which students can solve to give

$r_C = -0.855, 0.964, 7.56$ kpc [1.0]

Select the correct $r_C = 0.964$ for the final answer

because for the last data point $r \gg r_C$ [1.5]

Less accurate r_C :

Omit the last term,

$M_{DM}(r) \approx 4\pi\rho_0 r_C^2 r$ [1.0]

$r_C \approx \sqrt{\frac{1.74\times10^{10}M_\odot}{4\pi\times1.40\times\frac{10^8 M_\odot}{kpc^3}\times12.05kpc}} = 0.901\ kpc$ [1.0]

Part 2 Dark matter distribution

(D2.4) By comparing Equation (4) to a linear function, the central density ρ_0 could also be found by a linear fit. Plot an appropriate graph so that a linear fit can be used to find another value of ρ_0. Evaluate ρ_0 in units of $\frac{M_\odot}{kpc^3}$ (mark your graph as "D2.4"). If

you cannot find the value of r_C from the previous part, use $r_C = 3.2$ kpc as an estimate for this part. [19]

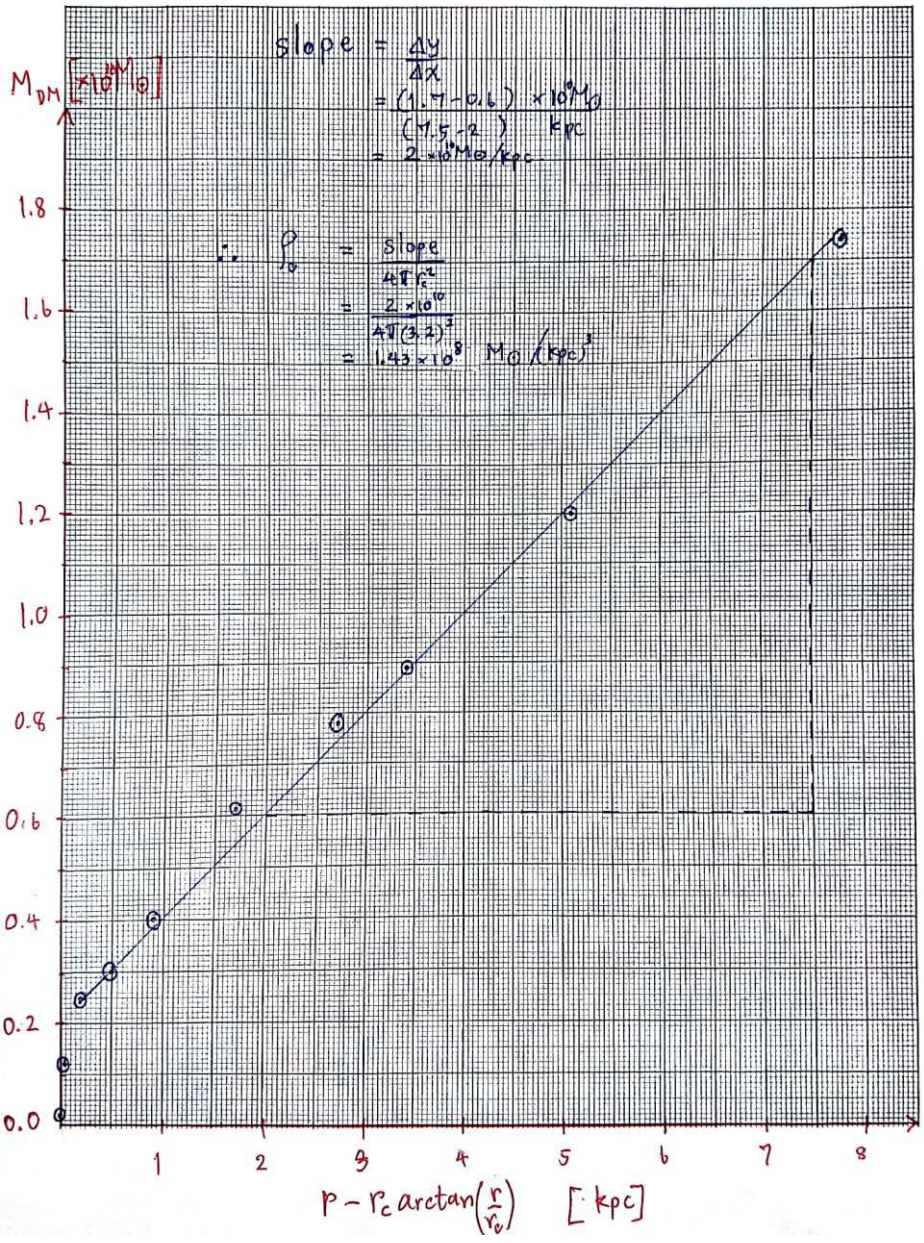
Solution 1:

$x = \left[r - r_c \arctan\left(\frac{r}{r_C}\right)\right]$

$y = M_{DM}(r)$

$x = \left[r - r_C \arctan(r / r_C)\right]$ (kpc)	$y = M_{\mathrm{DM}}(r)$ (kg)
0.010855	2.02×10^{8}
0.0803	1.18×10^{9}
0.2386	2.49×10^{9}
0.495	3.41×10^{9}
0.838	4.06×10^{9}
1.718	6.27×10^{9}
2.738	7.88×10^{9}
3.428	8.80×10^{9}
5.093	1.20×10^{10}
7.854	1.74×10^{10}

Graph Number D2.4: Solution 1



- Choose correct axes: 1.0, each correct axis get 0.5 mark
- Values correctly computed: 8.0 (0.4 for each value, 20 data points)
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.2 for each point, 20 data points)
  Values of x axis wrong due to previous wrong values get a maximum of 2.0 marks
- Credit for good visual linear fit: 1.0
- Correct value of slope: 1.0
- Correct value of ρ_0 : 1.0

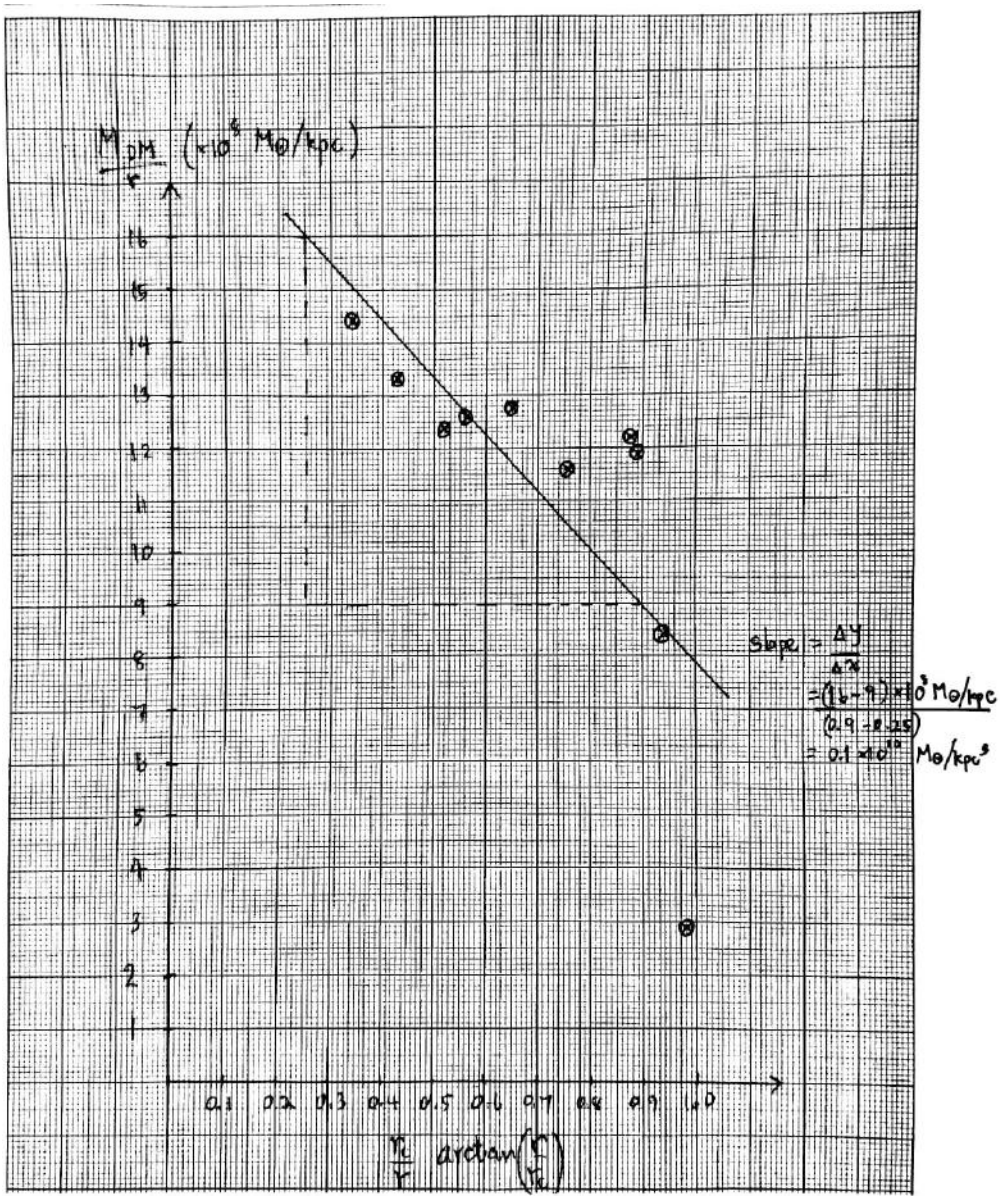
Solution 2:

$x = \frac{r_C}{r}\arctan\left(\frac{r}{r_C}\right)$

$y = M_{DM}(r)$

$x = \frac{r_C}{r}\arctan\left(\frac{r}{r_C}\right)$	$y = M_{\mathrm{DM}}(r)$ $(M_{\odot})$
0.984	2.89×10^{8}
0.943	8.40×10^{8}
0.886	1.19×10^{9}
0.822	1.22×10^{9}
0.760	1.16×10^{9}
0.649	1.28×10^{9}
0.562	1.26×10^{9}
0.517	1.24×10^{9}
0.436	1.33×10^{9}
0.348	1.44×10^{9}

Graph Number D2.4: Solution 2



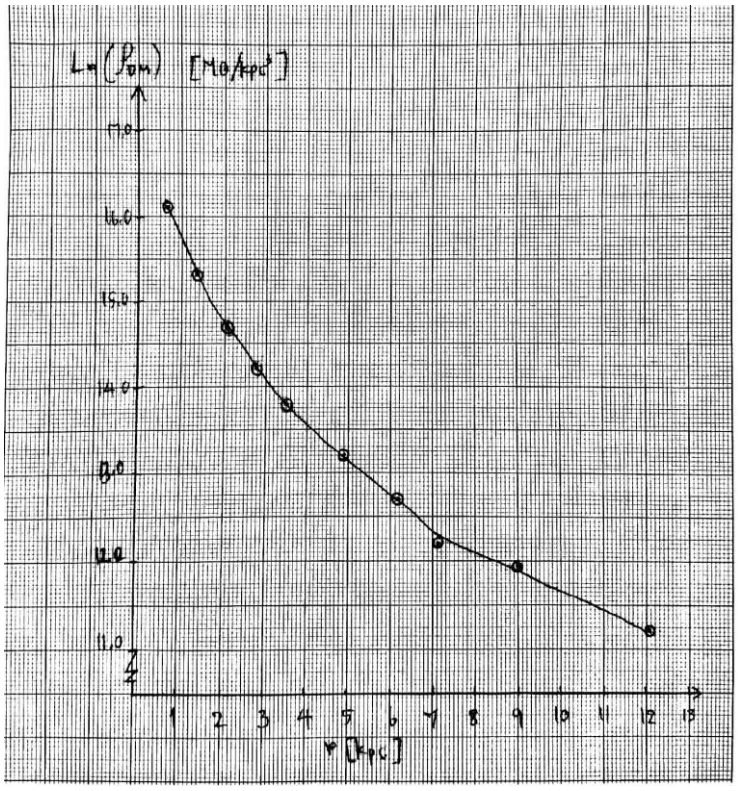
- Choose correct axes: 1.0, each correct axis get 0.5 mark
- Values correctly computed: 8.0 (0.4 for each value, 20 data points)
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.2 for each point, 20 data points)
  Values of x axis wrong due to previous wrong values gets a maximum of 2.0 marks
- Credit for good visual linear fit: 1.0
- Correct value of slope: 1.0
- Correct value of ρ_0 : 1.0

(D2.5) Compute logarithmic values of the dark matter density, ln $[\rho_{DM}(r)]$, and plot the distribution of the dark matter in the galaxy as a function of radius r on graph paper (mark your graph as "D2.5"). [12]

Solution 1: $r_c = 0.906\ kpc$ from D2.3

r (kpc)	ln $[\rho_{DM}(r)]$ $(\frac{M_\odot}{kpc^3})$
0.70	16.11
1.40	15.36
2.09	14.73
2.79	14.23
3.49	13.81
4.89	13.17
6.25	12.69
7.10	12.44
9.03	11.97
12.05	11.40

Graph Number D2.5: Solution 1



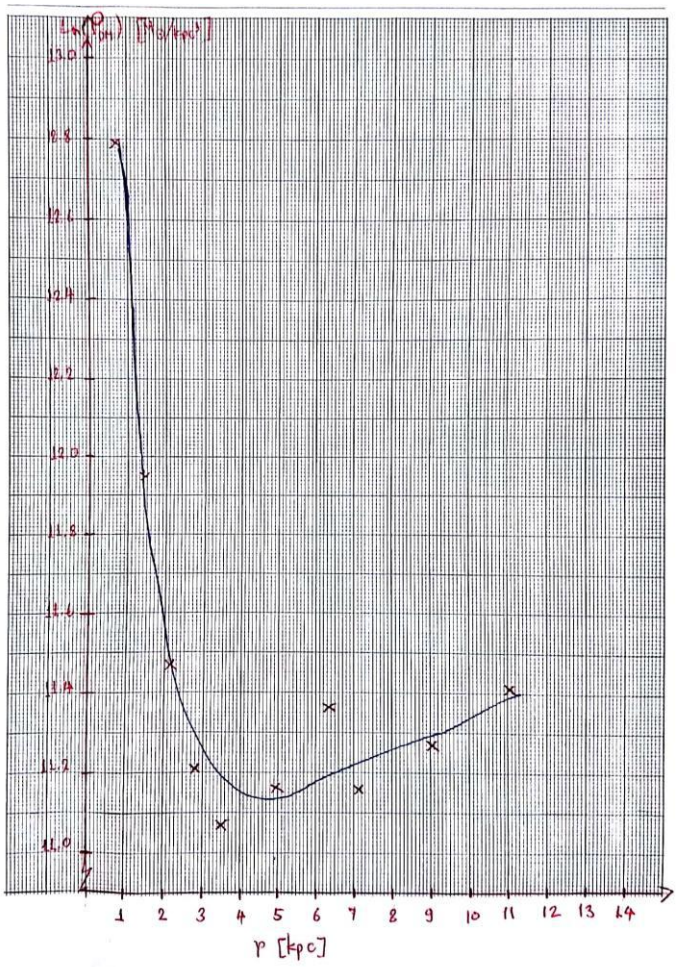
- Values correctly computed: 4.0 (0.4 for each values, 10 data points)

  Values of ln [ρ_{DM}(r)] wrong due to the wrong value of ρ_0 gets a maximum of 2.0 marks
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.4 for each point, 10 data points)
  Values of ln [ρ_{DM}(r)] wrong due to previous wrong values gets a maximum of 2.0 marks
- Smooth curve through points: 1.0

Solution 2: $r_c = 3.2\ kpc$

r (kpc)	ln [ρ_{DM}(r)] ($\frac{M_\odot}{kpc^3}$)
0.70	12.79
1.40	11.95
2.09	11.47
2.79	11.21
3.49	11.07
4.89	11.16
6.25	11.26
7.10	11.16
9.03	11.27
12.05	11.41

Graph Number D2.5 Solution 2



- Values correctly computed: 4.0 (0.4 for each values, 10 data points)
  Values of ln $[\rho_{DM}(r)]$ wrong due to the wrong value of ρ_0 gets a maximum of 2.0 marks
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.4 for each point, 10 data points)
  Values of ln $[\rho_{DM}(r)]$ wrong due to previous wrong values gets a maximum of 2.0 marks
- Smooth curve through points: 1.0