Instructions

1. The data analysis competition lasts for 5 hours and is worth a total of 150 points.

2. Dedicated IOAA **Summary Answer Sheets** are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding **Summary Answer Sheet**. On each Answer Sheet, please fill in

- Student's Code

3. **Graph Paper** is required for your solutions. On each Graph Paper sheet, please fill in

- Student's Code
- Question no.
- Graph no. and total number of graph paper sheets used.

4. There are **Answer Sheets** for carrying out detailed work/rough work. On each Answer Sheet, please fill in

- Student's Code
- Question no.
- Page no. and total number of pages.

5. Start each problem on a separate Answer Sheet. **Please write only on the printed side of the sheet. Do not use the reverse side.** If you have written something on any sheet which you do not want to be marked, cross it out.

6. Use as many mathematical expressions as you think may help the graders to better understand your solutions. The graders may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).

7. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, Graph Paper etc.), please put up your hand to signal the invigilator.

8. The beginning and end of the competition will be indicated by a long sound signal. Additionally, there will be a short sound signal fifteen minutes before the end of the competition (before the final long sound signal).

9. At the end of the competition you must stop writing immediately. Sort and put your **Summary Answer Sheets**, **Graph Papers**, and **Answer Sheets in one stack**. Put all other papers in another stack. You are not allowed to take any sheet of paper out of the examination area.

10. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

11. A list of constants is given on the next page.

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	$9.81\ \mathrm{ms}^{-2}$	
Obliquity of Ecliptic	$23°27'$	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass ($M_\leftmoon$)	7.35×10^{22} kg	**Moon**
Radius ($R_\leftmoon$)	1.74×10^{6} m	
Mean Earth-Moon distance	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	-12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude	4.80 mag	
Surface Temperature	5772 K	
Angular diameter at Earth	$30'$	
Orbital velocity in Galaxy	$220\ \mathrm{kms}^{-1}$	
Distance from Galactic center	8.5 kpc	
1 *au*	1.50×10^{11} m	**Physical constants**
1 *pc*	206265 au	
Gravitational constant (G)	$6.67 \times 10^{-11}\ \mathrm{Nm}^2\mathrm{kg}^{-2}$	
Planck constant (h)	6.62×10^{-34} Js	
Boltzmann constant (k_B)	$1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$	
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8}\ \mathrm{Wm}^{-2}\mathrm{K}^{-4}$	
Hubble constant (H_0)	$67.8\ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$	
Speed of light in vacuum (c)	$299792458\ \mathrm{ms}^{-1}$	
Magnetic Permeability of free space (μ_0)	$4\pi \times 10^{-7}\ \mathrm{Hm}^{-1}$	
1 Jansky (Jy)	$10^{-26}\ \mathrm{W\ m^{-2}Hz^{-1}}$	

$$\Delta \log_{10}(x) = \frac{\Delta x}{x\ ln10}$$

(D1) Dust and Young Stars in Star-forming Galaxies [75 points]

As a by-product of the star-forming process in a galaxy, interstellar dust can significantly absorb stellar light in ultraviolet (UV) and optical bands, and then re-emit in far-infrared (FIR), which corresponds to a wavelength range of 10-300 μm.

1.1. In the UV spectrum of a galaxy, the major contribution is from the light of the young stellar population generated in recent star-formation processes, thus the UV luminosity can act as a reliable tracer of the star-formation rate (SFR) of a galaxy. Since the observed UV luminosity is strongly affected by dust attenuation, extragalactic astronomers define an index called the *UV continuum slope* (β) to quantify the shape of the UV continuum:

$$f_\lambda = Q \cdot \lambda^\beta$$

where f_λ is the monochromatic flux of the galaxy at a given wavelength λ (in the unit of W m^{-3}) and Q is a scaling constant.

(D1.1.1) (6 points) AB magnitude is a specific magnitude system. The AB magnitude is defined as:

$$\mathrm{m_{AB}} = -2.5 \log \frac{f_\nu}{3631\ \mathrm{Jy}}$$

The AB magnitude of a typical galaxy is roughly constant in the UV band. What is the **UV continuum slope** of this kind of galaxy? (Hint: $f_\nu \Delta\nu = f_\lambda \Delta\lambda$)

(D1.1.2) (12 points) Table 1 presents the observed IR photometry results for a $z = 6.60$ galaxy called *CR7*. **Plot** the AB magnitude of *CR7* **versus the logarithm of the rest-frame** wavelength on graph paper and labelled as **Figure 1**.

(D1.1.3) (5 points) Calculate *CR7*'s UV slope, **plot** the best-fit UV continuum on *Figure 1* and make a comparison with the results you obtained in (D1.1.1). Is it dustier than the typical galaxy in (D1.1.1)? **Please answer with [YES] or [NO]**. (Hint: Express $\mathrm{m_{AB}}$ as a function of λ and m_{1600} , where m_{1600} is the AB magnitude at $\lambda_0 = 160$ nm (1600 Å))

Table 1. (Observed Frame) IR Photometry of CR7 at z = 6.60

Band	*Y*	*J*	*H*	*K*
Central Wavelength (μm)	1.05	1.25	1.65	2.15
AB Magnitude	24.71 ± 0.11	24.63 ± 0.13	25.08 ± 0.14	25.15 ±0.15

1.2. Under the assumption that dust grains in the galaxy absorb the energy of UV photons and re-emit it by blackbody radiation, the relation between the UV continuum slope (β), UV brightness (at 1600 Å) and FIR brightness could be established:

$$\mathrm{IRX} \equiv \log\left(\frac{F_{FIR}}{F_{1600}}\right) = S(\beta)$$

where F_{FIR} is the observed far-infrared flux and F_{1600} is the observed flux at rest-frame wavelength 160 nm (1600 Å) (The "flux" F_λ is **defined** as $F_\lambda = \lambda \cdot f_\lambda$). Table 2 presents 20 measurements of β, F_{FIR} and F_{1600} in nearby galaxies (Meurer et al. 1999).

Table 2. UV slope, flux and FIR flux of 20 nearby galaxies

Galaxy Name	UV Slope β	$\log(F_{1600}/10^{-3}\mathrm{Wm}^{-2})$	$\log(F_{FIR}/10^{-3}\mathrm{Wm}^{-2})$
NGC4861	-2.46	-9.89	-9.97
Mrk 153	-2.41	-10.37	-10.92
Tol 1924-416	-2.12	-10.05	-10.17
UGC 9560	-2.02	-10.38	-10.41
NGC 3991	-1.91	-10.14	-9.80
Mrk 357	-1.80	-10.58	-10.37
Mrk 36	-1.72	-10.68	-10.94
NGC 4670	-1.65	-10.02	-9.85
NGC 3125	-1.49	-10.19	-9.64
UGC 3838	-1.41	-10.81	-10.55
NGC 7250	-1.33	-10.23	-9.77
NGC 7714	-1.23	-10.16	-9.32
NGC 3049	-1.14	-10.69	-9.84
NGC 3310	-1.05	-9.84	-8.83
NGC 2782	-0.90	-10.50	-9.33
NGC 1614	-0.76	-10.91	-8.84
NGC 6052	-0.72	-10.62	-9.48
NGC 3504	-0.56	-10.41	-8.96
NGC 4194	-0.26	-10.62	-8.99
NGC 3256	0.16	-10.32	-8.44

(D1.2.1) (14 points) Based on the data given in Table 2, **plot** the IRX $-\beta$ diagram on graph paper and labelled as **Figure 2** and find a linear fit to the data. **Write down** your best-fit equation (i.e. IRX $= a \cdot \beta + b$) by the side of your diagram.

(D1.2.2) (6 points) Quantify the **dispersion** (in 'units' of dex, where $\mathbf{for\ example, \log(10^9) - \log(10^4) = 5\ dex}$) between the observed $\mathrm{IRX_{obs}}$ and predicted $\mathrm{IRX_{pred}}$ using the following equation:

$$\sigma = \sqrt{\frac{\sum(\Delta \mathrm{IRX}_i)^2}{N-1}}\ (\mathrm{unit: dex})\ where\ \Delta \mathrm{IRX}_i = \mathrm{IRX}_{i,\mathrm{obs}} - \mathrm{IRX}_{i,\mathrm{pred}}$$

1.3. Under the previous assumption of the energy transfer process, the ratio of F_{FIR} to F_{1600} can be expressed as:

$$\frac{F_{FIR}}{F_{1600}} \approx 10^{0.4A_{1600}} - 1$$

Where F_{1600} is the unattenuated flux and A_λ is the dust absorption (in magnitudes) as a function of wavelength λ.

(D1.3.1) (6 points) Express A_{1600} as a function of IRX.

(D1.3.2) (12 points) Based on Table 2 data and the function of $A_{1600}(IRX)$ you derived above, **plot** the $A_{1600} - \beta$ diagram on graph paper and label it as **Figure 3** and find a linear fit to the data. **Write down** your best-fit equation (i.e.$A_{1600} = a' \cdot \beta + b'$) by the side of your diagram.

(D1.3.3) (2 points) If your linear model in (D1.3.2) is correct, what is the expected **UV continuum slope** β_0 of a dust-free galaxy?

1.4. After establishing the local relation between UV continuum slope and IRX, we could probably test this empirical law in the high-redshift universe. In 2016, researchers obtained an Atacama Large Millimeter / submillimeter Array (ALMA) observation of CR7, and the FIR continuum corresponded to a 3σ upper limit of an FIR flux of 1.5×10^{-19} W m^{-2}.

(D1.4.1) (6 points) Calculate the **IRX of *CR7***. Is it an upper limit or lower limit?

Hint: here F_{1600} should be written in the form of:

$$F_{1600} = \lambda_0 \cdot f_{1600}$$

where $\lambda_0 = 160$ nm (1600 Å) and f_{1600} is the observed flux in the rest-frame

(D1.4.2) (6 points) Is the current observation long enough to show any deviation of *CR7* from the IRX$-\beta$ relationship you just derived in the local universe? **Please answer with [YES] or [NO] on the summary answer sheet, give the IRX difference and show the working used to calculate it on the answer sheet**.