(D2)Compact Object in a Binary System [75 points]

Astronomers discovered an extraordinary binary system in the constellation of Auriga during the course of the Apache Point Observatory Galactic Evolution Experiment (APOGEE). In these questions, you will try to analyze the data and recreate their discovery by yourself.

The research team is aiming to find compact stars in binary systems using the radial velocity (RV) technique. They examined archival APOGEE spectra of "single" stars and measured the variation of their RV. Among ~200 stars with the highest accelerations, researchers searched for periodic photometric variations in data from the All-Sky Automated Survey for Supernovae (ASAS-SN) that might be indicative of transits, ellipsoidal variations or starspots. After this process, they spotted a star named 2M05215658+4359220, with large variation in RV and photometric variability.

2.1. The following table presents the radial velocity measurements of 2M05215658+4359220 during three epochs of APOGEE spectroscopic observation. Here we assume the variation of its RV is due to the existence of an unseen companion. The proper motion of the stars can be ignored.

Table 3. APOGEE Radial Velocity Measurements of 2M05215658+4359220

MJD	RV (km/s)	Uncertainty (km/s)
56204.9537	-37.417	0.011
56229.9213	34.85	0.01
56233.8732	42.57	0.01

(D2.1.1) (6 points) Use the data to obtain a rough estimate of the apparent **maximum acceleration:**

$a_{max} = \frac{\Delta RV}{\Delta t}\Big|_{max}$, unit: km/s/day

of the star.

(D2.1.2) (9 points) Estimate the **mass** of its unseen companion with reasonable models.

2.2. After discovering this peculiar star, astronomers conducted follow-up observations using the 1.5m Tillinghast Reflector Echelle Spectrograph (TRES) at the Fred Lawrence Whipple Observatory (FLWO) located on Mt. Hopkins in Arizona, USA. The following table presents the RV measurements using this instrument:

Table 4. TRES Radial Velocity Measurements of 2M05215658+4359220

MJD	RV (km/s)	Uncertainty (km/s)
58006.9760	0	0.075
58023.9823	-43.313	0.075
58039.9004	-27.963	0.045
58051.9851	10.928	0.118
58070.9964	43.782	0.075
58099.8073	-30.033	0.054
58106.9178	-42.872	0.135
58112.8188	-44.863	0.088
58123.7971	-25.81	0.115

58136.6004	15.691	0.146
58143.7844	34.281	0.087

(D2.2.1) (14 points) **Plot** the diagram of RV variation (measured with TRES) versus time on your graph paper as Figure 4. Connect all your data points with a **sinusoidal function**. **Estimate** the orbital period (P_{orb}) and radial velocity semi-amplitude (K) from your plot.

(D2.2.2) (4 points)If the star is rotating in circular orbit, show the **minimum value** of the orbital radius (r_{orb}) of the star in unit of both $R_\odot$ and au.

(D2.2.3) (7 points)The mass function of a binary system is defined as:

$$f(M_1, M_2) = \frac{M_2^3 \sin^3 i_{orb}}{(M_1 + M_2)^2}$$

where the subscript "1" represents the primary star and "2" represents its companion. The parameter i_{orb} is the orbital inclination of the binary system. Calculate the **mass function of this system** in unit of $M_\odot$.

2.3. Based on a detailed analysis on APOGEE, TRES spectra and GAIA parallax measurements, astronomers derived the following stellar parameters:

Table 5. Selected Physical Properties of 2M05215658+4359220

Effective Temperature T_{eff} (K)	Surface Gravity $\log g$ (cm/s^2)	Parallax π (mas)	Radial Rotation Velocity $v \sin i_{rot}$ (km/s)	Bolometric Flux F (J s^{-1}m^{-2})
4890 ± 130	2.2 ± 0.1	0.272±0.049	14.1±0.6	(1.1±0.1)×10^{-12}

Photometric observations indicate that the period of its light curve is identical to orbital period, thus we may assume that rotation period satisfies $P_{rot} = P_{orb} \equiv P$, and the inclination satisfies $i_{orb} = i_{rot} \equiv i$.

(D2.3.1) (16 points)**Calculate** the sine of inclination angle ($\sin i$), radius (R_1, in unit of $R_\odot$), luminosity (L_1, in unit of $L_\odot$), as well as mass (M_1, in unit of $M_\odot$) of the visible star. Please **include** the uncertainty into your results.

(D2.3.2) (4 points)**Choose** the correct type of this star from the following options: (1) Blue Giant (2) Blue Dwarf (3) Red Giant (4) Red Dwarf (5) Main-Sequence Star (6) White Dwarf.

(D2.3.3) (10 points)Based on the mass function $f(M_1, M_2)$of binary system, **plot** out the rough relation of M_2 (as vertical axis) and M_1 (as horizontal axis) on your graph paper as Figure 5. Here you should adopt the $\sin i$, $\sin i + \Delta \sin i$and $\sin i - \Delta \sin i$derived in (D2.3.1), and plot out three functions **respectively**.

(D2.3.4) (5 points)Draw a vertical **shadowed region** of $[M_1 - \Delta M_1, M_1 + \Delta M_1]$, as well as two horizontal **dashed line** showing the maximum mass of white dwarf and neutron star, on your *Figure 5*. What is the possible mass of the invisible companion, and what kind of celestial object could it be?

Solution:

2.1. APOGEE RV Measurements[15 marks]

(D2.1.1)[6 marks]Table 3 gives 3 epochs of RV measurements from MJD 56204 to 56233, and the maximum acceleration occurred during the first and the second observation:

$$a_{max} = \frac{RV_2 - RV_1}{t_2 - t_1} = 2.89\ \mathrm{km/(s \cdot d)} \ldots\ldots (15)$$

(D2.1.2)[9 marks]An easy approach to estimate the companion mass is consider the force exerted on the star:

$$F = M_1 a = \frac{GM_1M_2}{r^2} \Rightarrow M_2 = \frac{ar^2}{G} \ldots\ldots (16)$$

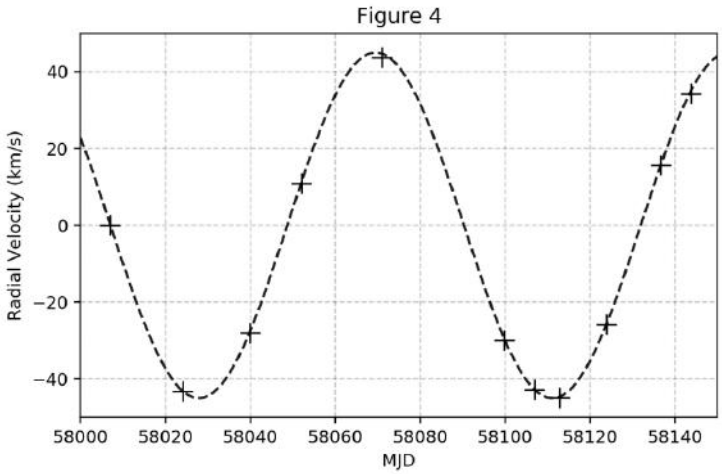
Therefore, we need to estimate a and r properly. Here we can assume the acceleration of the primary star as$a = a_{max}$, and the distance between the two star $r = (v_2 - v_1) \cdot (t_2 - t_1)/2$, and we can derive:

$$M_2 \sim \frac{(v_2 - v_1)^3 (t_2 - t_1)}{4G} = 1.5\ M_\odot \ldots\ldots (17)$$

(*Note: This is only a rough estimate. If examinees derive a* M_2*within the range of* $0.5M_\odot \sim 3\ M_\odot$[4 marks]*with clear and proper assumption*[5 marks]*, then his/her solution should be considered correct.)*

2.2. TRES RV Measurements[25 marks]

(D2.2.1)[14 marks]Given the data in Table 4, we can plot out all the TRES RV measurements, with a sinusoidal function connecting all the points:



Grading standard of Figure 4: (1) [1 mark]The figure is large enough on the graph paper (larger than 50% of usable space); (2) [2 marks]horizontal axis is MJD, vertical axis is RV (in unit of km/s), and both axes are plotted in linear scale; (3) [1 mark]Ticks on both axes are properly labeled with suitable spacing; (4) [6 marks]Data points and sinusoidal function are plotted properly.

The best-fit parameters of period an RV semi-amplitude are:

$$P_{orb} = 83.26 \pm 0.02\ \mathrm{d};\ \ K = 45.02 \pm 0.03\ \mathrm{km/s} \ldots\ldots (18)$$

(error bars are reported here for benefit of jury. They do not carry any points)

[2 marks for each parameter]

Answers within the range of $P_{orb} \in [81, 85]$d and $K \in [44, 46]$ km/s could be considered correct.

(D2.2.2)[4 marks]The orbital radius satisfies that:

$$2\pi R_{orb} = P_{orb} \cdot \frac{K}{\sin i_{orb}} \Rightarrow R_{orb} > \frac{P_{orb}K}{2\pi} = 74\ R_{\odot} = 0.34\ \text{AU} \ldots\ldots (19)$$

(D2.2.3)[7 marks]The mass function of a binary system is defined as:

$$f(M_1, M_2) = \frac{M_2^3 \sin^3 i_{orb}}{(M_1 + M_2)^2} \ldots\ldots (20)$$

This mass function is actually derived from Newton's law (force balance):

$$\frac{GM_1M_2}{D^2} = M_1 \left(\frac{2\pi}{P}\right)^2 R_{orb} \ldots\ldots (21)$$

And the relation between orbital radius and binary separation (D):

$$R_{orb} = \frac{M_2}{M_1 + M_2} D = \frac{P_{orb}K}{2\pi \sin i_{orb}} \ldots\ldots (22)$$

Therefore, Equation 20 could be rewritten as:

$$f(M_1, M_2) = \frac{M_2^3 \sin^3 i_{orb}}{(M_1 + M_2)^2} = \frac{K^3 P_{orb}}{2\pi G} \approx 0.79\ M_{\odot} \ldots\ldots (23)$$

[Equation (21), (22): 2 marks respectively; Equation (23) and the result: 3 marks]

2.3. Stellar Parameters[35 marks]

(D2.3.1)[16 marks]The first (and the most direct) thing we could solve from the given parameter table is the distance of this system, which is:

$$d = \frac{1}{\pi} = 3.68\ \text{kpc} \ldots\ldots (24)$$

And the uncertainty of d should be:

$$\frac{\Delta d}{d} = \sqrt{\left(-\frac{\Delta \pi}{\pi}\right)^2} \Rightarrow \Delta d = \frac{\Delta \pi}{\pi} \cdot d = 0.66\ \text{kpc} \ldots\ldots (25)$$

Therefore, the luminosity of the star should be:

$$L_1 = 4\pi d^2 \cdot F = 464\ L_{\odot} \ldots\ldots (26)$$

And the uncertainty is:

$$\frac{\Delta L_1}{L_1} = \sqrt{\left(\frac{\Delta F}{F}\right)^2 + \left(\frac{2\Delta d}{d}\right)^2} \Rightarrow \Delta L_1 = 173\ L_{\odot} \ldots\ldots (27)$$

By using Stefan-Boltzmann law, we can calculate the radius of the star:

$$R_1 = \sqrt{\frac{L_1}{\sigma T^4 \cdot 4\pi}} = 30\ R_\odot \ldots\ldots (28)$$

With an uncertainty of:

$$\Delta R_1 = R_1 \cdot \sqrt{\left(-2\frac{\Delta T}{T}\right)^2 + \left(\frac{1}{2}\frac{\Delta L_1}{L_1}\right)^2} = R_1 \cdot \sqrt{\left(-2\frac{\Delta T}{T}\right)^2 + \left(\frac{1}{2}\frac{\Delta F}{F}\right)^2 + \left(\frac{\Delta d}{d}\right)^2}$$
$$= 6\ R_\odot \ldots\ldots (29)$$

Combined with the observed rotational velocity, we can calculate the sine of inclination angle:

$$\sin i = \frac{P \cdot v \sin i}{2\pi R_1} = 0.77 \ldots\ldots (30)$$

And the uncertainty should be:

$$\Delta \sin i = \sin i \cdot \sqrt{\left(-\frac{\Delta R_1}{R_1}\right)^2 + \left[\frac{\Delta(v \sin i)}{v \sin i}\right]^2} = 0.15 \ldots\ldots (31)$$

Now we can calculate the mass M_1. First of all, we need to translate the surface gravity from log scale to linear scale:

$$g = 10^{\log g} = 1.58\ \mathrm{m/s^2} \ldots\ldots (32)$$

With an uncertainty of:

$$\Delta g = g \ln 10 \cdot \Delta(\log g) = 0.36\ \mathrm{m/s^2} \ldots\ldots (33)$$

Therefore, M_1 should be:

$$M_1 = \frac{g R_1^2}{G} = 5.2\ M_\odot \ldots\ldots (34)$$

And the uncertainty is:

$$\Delta M_1 = M_1 \sqrt{\left(\frac{\Delta g}{g}\right)^2 + \left(2\frac{\Delta R_1}{R_1}\right)^2} = 2.3\ M_\odot \ldots\ldots (35)$$

[For each quantity (sin*i*, R_1, M_1, L_1):

deduction: 2 marks; exact value: 1 mark; uncertainty: 1 mark]

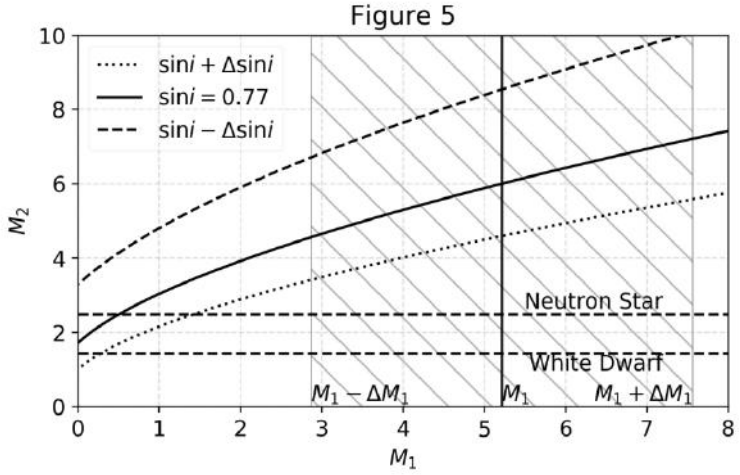
(D2.3.2)[4 marks]From the temperature and the luminosity, we can find that the primary star should be a (3) Red Giant.

(D2.3.3)[10 marks]Since we have derived the binary system mass function, with a given range of M_1, we can calculate the corresponding range of M_2 and hence make a plot of $M_2 - M_1$ (Figure 5). Detailed standards of grading the plot are in the solution of (D2.3.4).

(D2.3.4)[5 marks]The vertical shadowed zone should start at 2.9 $M_\odot$ and end at 7.5 $M_\odot$, while the maximum mass of white dwarf is $1.4M_\odot$and the maximum mass of neutron star should be around 2.4~2.6 $M_\odot$(here we adopt 2.5 $M_\odot$ on the plot). From Figure 5, we can readout that the possible mass of the companion is roughly $6^{+4}_{-2.5}M_\odot$. Since we didn't receive any optical radiation from the companion, as well as M_2 has already exceeded the maximum mass of neutron star, the invisible companion in this binary system could be a **stellar-mass black hole**, or some unknown compact object that current theory hasn't predicted.

[Possible Mass: 2 marks; Type: 2 marks]

Figure 5



(Note: Examinee's answer is not required to give an uncertainty range of M_2: if the answer is within the possible range of $6^{+4}_{-2.5}M_\odot$, the answer will be considered correct. In addition to this, examinee don't need to include "unknown compact object" in his/her answer while addressing the possible type of the companion: an answer like "black hole" or "stellar-mass black hole" is enough for receiving full grades.)

Grading standard of Figure 5: (1) [1 mark]The figure is large enough on the graph paper (larger than 50% of usable space); (2) [2 marks]horizontal axis is M_1, vertical axis is M_2 (both in unit of $M_\odot$), and both axes are plotted in linear scale; (3) [1 mark]Ticks on both axes are properly labeled with suitable spacing; (4) [3 mark]All of the three mass functions are plotted properly with obvious and correct labels/legends; (5) [1 mark]Vertical shadowed region of possible M_1 range is plotted correctly; (5) [2 marks]Two horizontal dashed lines are plotted at correct positions with clear and correct labels.