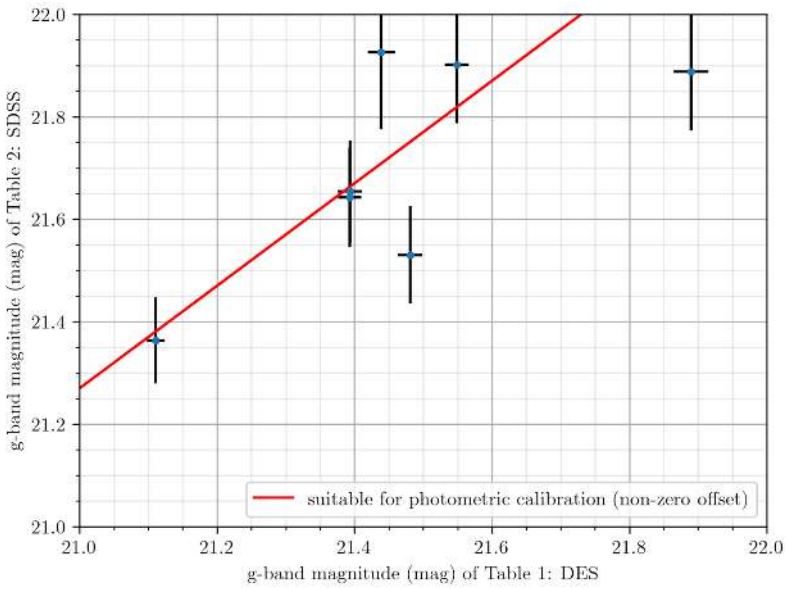


The student should identify the stars that are closest to the one-to-one line (x = y) between surveys. This could be with a zero offset, or a non-zero offset. The **two** stars that would be suitable for photometric calibration with zero offset are the ones with ID's **8** and **9** of Table 1 (DES). The **four** stars that would be suitable for photometric calibration with non-zero offset are the ones with ID's **2, 3, 4 and 6** of Table 1 (DES).

D2. Shapley Hypothesis (75 points)

Globular clusters are one of the oldest components of galaxies. About a century ago, Harlow Shapley studied the distribution of globular clusters in the Milky Way in order to determine the distance from the Sun to the Galactic Centre, with the hypothesis that globular clusters were symmetrically distributed around the Galactic Centre. The table below shows the positions and distance moduli of a few known globular clusters in the Milky Way. The first three columns in the table show the cluster name, galactic longitude (l), and galactic latitude (b). The fourth column shows the distance modulus (i.e. the difference between the apparent and absolute magnitude), for which the values are extinction-corrected. Based on the data in the table:

Name	l (degrees)	b (degrees)	Distance modulus (mag)
NGC 6522	1.025	-3.926	14.3
NGC 6401	3.450	3.980	14.4
NGC 6342	4.898	9.725	14.5
NGC 6553	5.253	-3.029	13.6
NGC 6440	7.729	3.801	14.6
Ter 12	8.358	-2.101	13.6
VW-CL160	10.151	0.302	14.2
2MASS-GC01	10.471	0.100	12.6
NGC 6517	19.225	6.762	14.8
NGC 6402	21.324	14.804	14.8
NGC 6712	25.354	-4.318	14.3
NGC 6426	28.087	16.234	16.6
NGC 5466	42.150	73.592	16.0
NGC 7089	53.371	-35.770	15.3
NGC 288	151.285	-89.380	14.8
NGC 2298	245.629	-16.006	15.0
NGC 4590	299.626	36.051	15.1
NGC 4372	300.993	-9.884	13.8
NGC 362	301.533	-46.247	14.7
BH 140	303.171	-4.307	13.4
NGC 5927	326.604	4.860	14.6
Patchick 126	340.381	-3.826	14.5
NGC 5897	342.946	30.294	15.5
NGC 6380	350.182	-3.422	14.9
Djor 1	356.675	-2.484	15.0

(a) (20 points) Calculate the distance (in parsecs) of each globular cluster from the Sun as well as their cartesian coordinates (x,y,z). The x-axis points to the Galactic Centre and the y-axis points in the direction of galactic rotation. The system is right-handed.

Solution: The first step is to convert the extinction corrected distance moduli (DM) of all globular clusters to distance (d), in parsecs:

$$DM = (m - M) - Av = 5 \cdot \log(d) - 5 \quad \Rightarrow \quad d = 10^{(DM+5)/5}\,\text{pc}$$

So, it follows to calculate the cartesian coordinates (x, y, z) of the globular clusters with respect to the Sun, using the galactic coordinates (longitude l and latitude b). The x-axis points to the Galactic Centre, the y-axis points to direction of galactic rotation and the z-axis is perpendicular to the galactic disk, and points in the direction antiparallel to the angular momentum. The conversion of coordinates should be done as follows:

$$x = d \cdot \cos(l) \cdot \cos(b) \quad , \quad y = d \cdot \sin(l) \cdot \cos(b) \quad , \quad z = d \cdot \sin(b)$$

The table below shows the calculated values of d, x, y, and z for all globular clusters.

Name	d (pc)	x (pc)	y (pc)	z (pc)
NGC 6522	7244.36	7226.20	129.29	-496.01
NGC 6401	7585.78	7553.77	455.39	526.52
NGC 6342	7943.28	7800.55	668.47	1341.77
NGC 6553	5248.07	5218.73	479.81	-277.32
NGC 6440	8317.64	8223.94	1116.16	551.39
Ter 12	5248.07	5188.85	762.34	-192.40
VVV-CL 160	6918.31	6809.92	1219.29	36.47
2MASS-GC01	3311.31	3256.16	601.79	5.78
NGC 6517	9120.11	8551.60	2982.17	1073.85
NGC 6402	9120.11	8213.73	3206.36	2330.31
NGC 6712	7244.36	6528.00	3093.30	-545.44
NGC 6426	20892.96	17697.53	9444.44	5840.86
NGC 5466	15848.93	3319.16	3004.35	15203.48
NGC 7089	11481.54	5558.08	7476.05	-6711.34
NGC 288	9120.11	-86.55	47.41	-9119.57
NGC 2298	10000.00	-3966.46	-8755.80	-2757.38
NGC 4590	10471.29	4185.03	-7359.22	6162.41
NGC 4372	5754.40	2919.15	-4859.63	-987.77
NGC 362	8709.64	3150.05	-5133.77	-6291.21
BH 140	4786.30	2611.38	-3995.02	-359.45
NGC 5927	8317.64	6919.31	-4561.75	704.68
Patchick 126	7943.28	7465.49	-2661.12	-530.03
NGC 5897	12589.25	10392.20	-3187.93	6350.49
NGC 6380	9549.93	9393.28	-1625.54	-570.03
Djor 1	10000.00	9973.79	-579.45	-433.40

(b) (20 points) From the given data, estimate the distance from the Sun to the centre of the distribution of globular clusters and its uncertainty.

Solution: In order to estimate the distance from the Sun to the centre of the distribution and its uncertainty, we first need the mean values of each coordinate $\bar{x}$, $\bar{y}$ and $\bar{z}$, as well as the standard deviations in each axis, σ_x, σ_y and σ_z. The calculations of the mean and standard deviation for the x, y, and z coordinates are as follows:

Mean in the three axes:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i, \quad \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i, \quad \bar{z} = \frac{1}{N}\sum_{i=1}^{N} z_i$$

$$\bar{x} = 6164.1 \text{ pc}, \quad \bar{y} = -321.3 \text{ pc}, \quad \bar{z} = 434.3 \text{ pc}$$

Standard deviations in the three axes:

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}, \quad \sigma_y = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N-1}}, \quad \sigma_z = \sqrt{\frac{\sum_{i=1}^{N}(z_i - \bar{z})^2}{N-1}}$$

$$\sigma_x = 4057.7\,\text{pc} \quad , \quad \sigma_y = 4196.4\,\text{pc} \quad , \quad \sigma_z = 4682.6\,\text{pc}$$

Consequently, we obtain the uncertainties associated to the mean in each axis:

Uncertainties of means in the three axes:

$$\delta\bar{x} = \frac{\sigma_x}{\sqrt{N}}, \quad \delta\bar{y} = \frac{\sigma_y}{\sqrt{N}}, \quad \delta\bar{z} = \frac{\sigma_z}{\sqrt{N}}$$

$$\delta\bar{x} = 811.54\text{ pc}, \quad \delta\bar{y} = 839.28\text{ pc}, \quad \delta\bar{z} = 936.52\text{ pc}$$

Finally, the distance D, in parsecs, from the Sun to the centre of the distribution of globular clusters is estimated as:

$$D = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}$$
$$D = \sqrt{(6164.1)^2 + (-321.3)^2 + (434.3)^2} = 6187.73\text{ pc}$$

And, to estimate its uncertainty δD, we should perform an error propagation calculation:

$$(\delta D)^2 = \left(\frac{\partial D}{\partial \bar{x}}\right)^2 \cdot (\delta\bar{x})^2 + \left(\frac{\partial D}{\partial \bar{y}}\right)^2 \cdot (\delta\bar{y})^2 + \left(\frac{\partial D}{\partial \bar{z}}\right)^2 \cdot (\delta\bar{z})^2 \Rightarrow$$
$$(\delta D)^2 = \left[\left(\frac{\bar{x}}{D} \cdot \delta\bar{x}\right)^2 + \left(\frac{\bar{y}}{D} \cdot \delta\bar{y}\right)^2 + \left(\frac{\bar{z}}{D} \cdot \delta\bar{z}\right)^2\right] \Rightarrow$$
$$\delta D = \frac{1}{6187.73} \cdot \left[(6164.1 \cdot 811.54)^2 + (-321.3 \cdot 839.28)^2 + (434.3 \cdot 936.52)^2\right]^{1/2} \Rightarrow$$
$$\delta D = 812.28\,\text{pc}$$

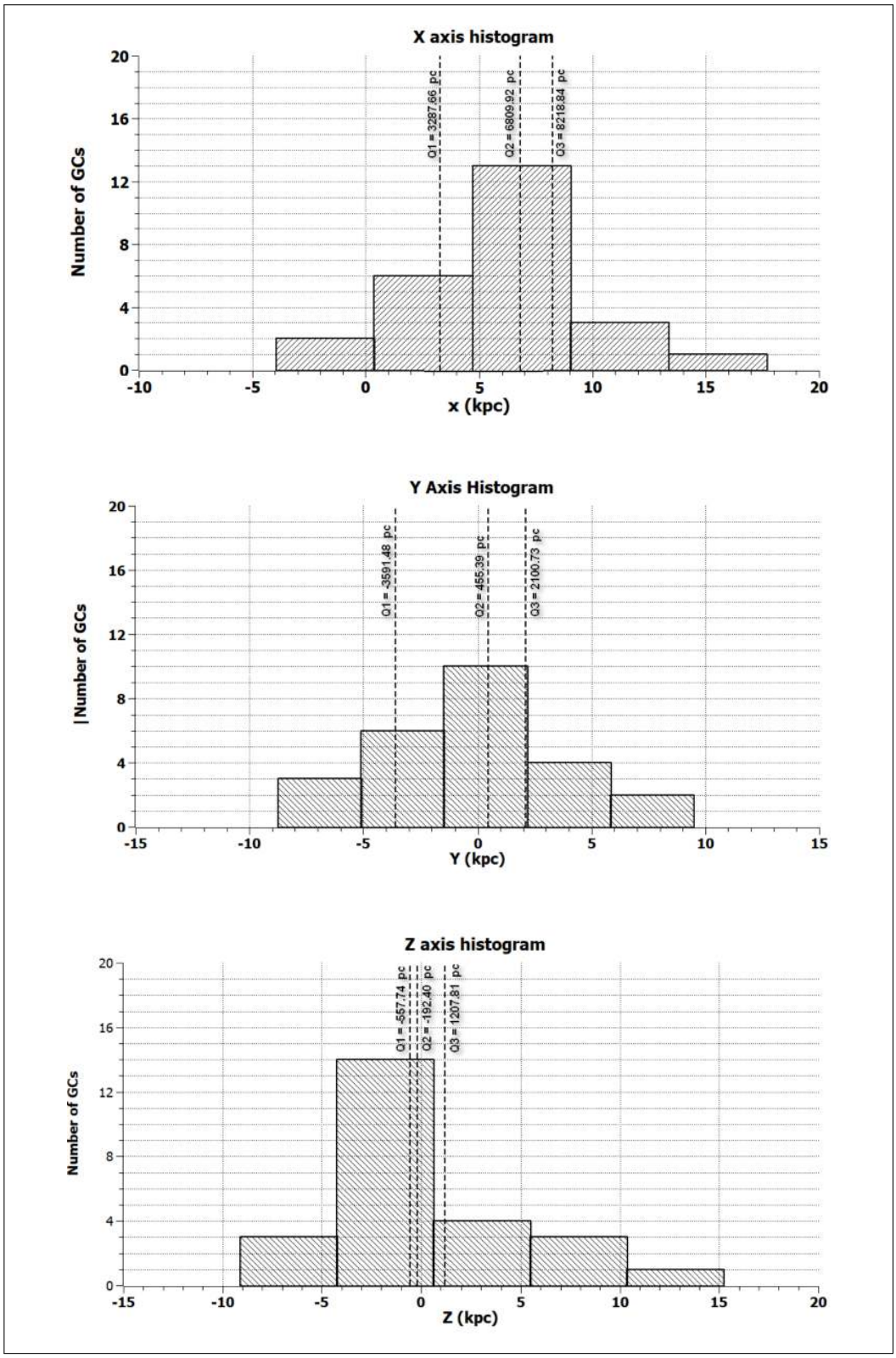
(c) (30 points) To test the validity of Shapley's hypothesis that globular clusters are symmetrically distributed around the Galactic Centre, make histograms with five bins (i.e. sort the data and divide them into five equally-sized intervals) for each of the distributions in the x, y, and z directions. Mark the value of the quartiles (Q_1, Q_2, Q_3) of the three distributions with solid lines on the histograms.

Hint: The three quartiles divide the sorted sample into four sections, each containing 25% of the data, with the second and third sections representing the interquartile range.

Solution:
The table below shows the calculated values of the quartiles for each distribution.

Axis	Q1 (pc)	Q2 (pc)	Q3 (pc)
x	3287.66	6809.92	8218.84
y	-3591.48	455.39	2100.73
z	-557.74	-192.40	1207.81

The expected histograms for each axis are as follows:


X axis histogram
Q1 = 3287.66 pc
Q2 = 6809.92 pc
Q3 = 8218.84 pc
Number of GCs
x (kpc)
Y Axis Histogram
Q1 = -3591.48 pc
Q2 = 455.39 pc
Q3 = 2100.73 pc
Number of GCs
Y (kpc)
Z axis histogram
Q1 = -557.74 pc
Q2 = -192.40 pc
Q3 = 1207.81 pc
Number of GCs
Z (kpc)

(d) (5 points) Using the quartiles, calculate the symmetry factor value for the three distributions as given by:

$$\Phi_x = \frac{|Q_{1,x} + Q_{3,x} - 2Q_{2,x}|}{Q_{3,x} - Q_{1,x}}, \Phi_y = \frac{|Q_{1,y} + Q_{3,y} - 2Q_{2,y}|}{Q_{3,y} - Q_{1,y}}, \Phi_z = \frac{|Q_{1,z} + Q_{3,z} - 2Q_{2,z}|}{Q_{3,z} - Q_{1,z}}$$

Classify the three distributions in the x, y, and z directions based on their calculated symmetry factor values, according to the table shown below. Hence, on the answer sheet, write True (**T**) if the analysed sample follows Shapley's hypothesis or False (**F**) otherwise.

Symmetry factor value	**Symmetry type**
$0.0 \leqslant \Phi \leqslant 0.1$	symmetrical
$0.1 < \Phi \leqslant 0.2$	quasi-symmetrical
$\Phi > 0.2$	asymmetrical

Solution:

F (False). The analysed sample does not follow Shapley's hypothesis, because all distributions are asymmetrical. See table below for the values of calculated symmetry factors for each distribution.

Axis	Φ	**Symmetry Type**
x	0,429	asymmetrical
y	0,422	asymmetrical
z	0,588	asymmetrical