(OP01) Hypothesized Planet in the Solar System [25 marks]

For this question use sky map Map-OP01. This is an angle preserving projection of the sky.

The sky projected on the planetarium dome is as seen from the solar system barycentre. The Earth's orbit as seen from this position has been marked in green both on the projected sky and in sky map Map-OP01. The first 3 minutes are for dark adaptation and familiarization of the sky.

After first 3 minutes, four Trans-Neptunian Objects (TNOs) labelled O1, O2, O3 and O4 will begin to appear on the dome. The brightness of the TNOs has been enhanced to make them visible. No other solar system object is visible in the projected sky. Time has been sped up to make their movements noticeable.

We will assume that the orbital plane of a hypothesized new planet has an inclination equal to the mean of the inclinations of the orbits of these 4 TNOs.

(OP01.1) Arrange the 4 TNOs in ascending order of the lengths of their semi-major axes and write their names in the appropriate boxes in the Summary Answersheet. Show measurements in the working sheet to justify your answer. **8**

Solution:

The orbits of the 4 TNOs are elliptical, which can be noticed from careful observations of their speed. Hence, to estimate lengths of their semi-major axes it is essential to measure the time period of one complete revolution of each TNO. The time periods of the planets are

O1: 40 sec, O2: 30 sec, O3: 48 sec, O4: 34 sec

Hence, the correct ascending order is: O2 < O4 < O1 < O3.

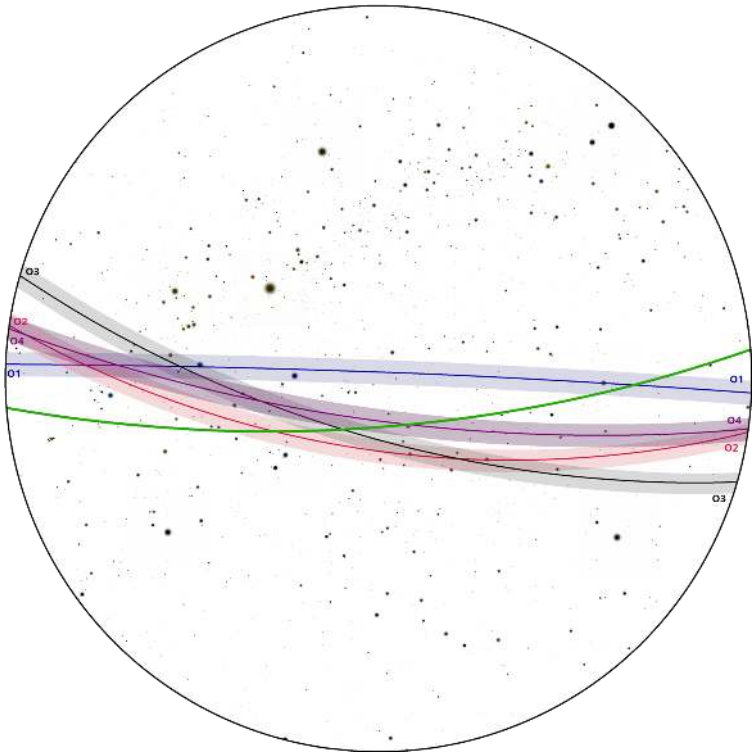
Credit of 1.0 for every pairwise correct order, and 2.0 extra for fully correct order.

O1 < O2 < O3 < O4	3.0		O2 < O1 < O3 < O4	4.0
O1 < O2 < O4 < O3	4.0		O2 < O1 < O4 < O3	5.0
O1 < O3 < O2 < O4	2.0		O2 < O3 < O1 < O4	3.0
O1 < O3 < O4 < O2	1.0		O2 < O3 < O4 < O1	4.0
O1 < O4 < O2 < O3	3.0		O2 < O4 < O1 < O3	8.0
O1 < O4 < O3 < O2	2.0		O2 < O4 < O3 < O1	5.0
O3 < O1 < O2 < O4	1.0		O4 < O1 < O2 < O3	4.0
O3 < O2 < O4 < O1	3.0		O4 < O3 < O1 < O2	2.0
O3 < O1 < O4 < O2	0.0		O4 < O1 < O3 < O2	3.0
O3 < O2 < O1 < O4	2.0		O4 < O2 < O1 < O3	5.0
O3 < O4 < O1 < O2	1.0		O4 < O2 < O3 < O1	4.0
O3 < O4 < O2 < O1	2.0		O4 < O3 < O2 < O1	3.0

(OP01.2) Trace at least 50% of the visible trajectories for each of the 4 TNOs on the map Map-OP01. From these traces find the inclination angles, i_1, i_2, i_3, i_4, of the 4 TNOs, respectively, with respect to the Ecliptic. Hence obtain the best estimate of the inclination, i_{hp}, of the plane of the orbit of the hypothesized planet. **17**

Solution:

The orbits of the TNOs can be drawn by marking a few important constellations and noticing the fact that some prominent or important stars lie along the TNOs' paths. Also the intersection points of each TNO trajectory with respect to the Earth's orbit (i.e. the Ecliptic) lie near prominent stars.



Since this map has an angle preserving (Stereoscopic) projection, measuring the angles directly using a protractor at the intersection points gives the inclination of the orbits with respect to the Ecliptic.

The measured values of inclination are:

- $i_1 = 18°$
- $i_2 = 14°$
- $i_3 = 20°$
- $i_4 = 12°$

The average inclination $\boxed{i_{\text{hp}} = 16°}$.

For each TNO:

If more than 50% of the visible orbit is traced at all	0.5

If more than 50% of the traced orbit lies within the tolerance band	1.5

Between 25% to 50%	1.0

Marks for the correctness of trace will be given only if 50% of the visible orbit is traced at all.

Measurement of the inclination angles within $\pm 2°$	2.0

Measurement of the inclination angles within $\pm 4°$	1.0

Marks for angle will be given only if more than 25% of the traced orbit lies inside the tolerance band.

Averaging of the angles	1.0