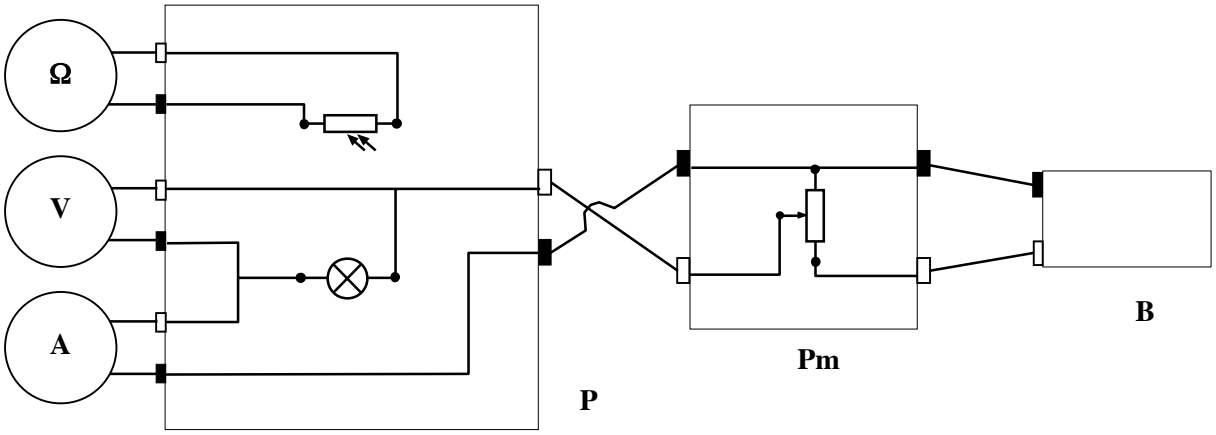
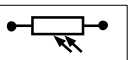
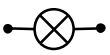
백열전구의 빛으로 구하는 플랑크 상수
풀이

과제 1

아래 상자 안과 상자 사이의 전기 연결을 그리시오.



광저항	
백열전구	
전위차계	
빨간 소켓	
검은 소켓	

Ω	**저항계**
V	**전압계**
A	**전류계**
P	**받침대**
Pm	**전위차계**
B	**전지**

과제 2

a)

$t_0 =$ 24 ℃	$T_0 =$ 297 K	$\Delta T_0 =$ 1 K

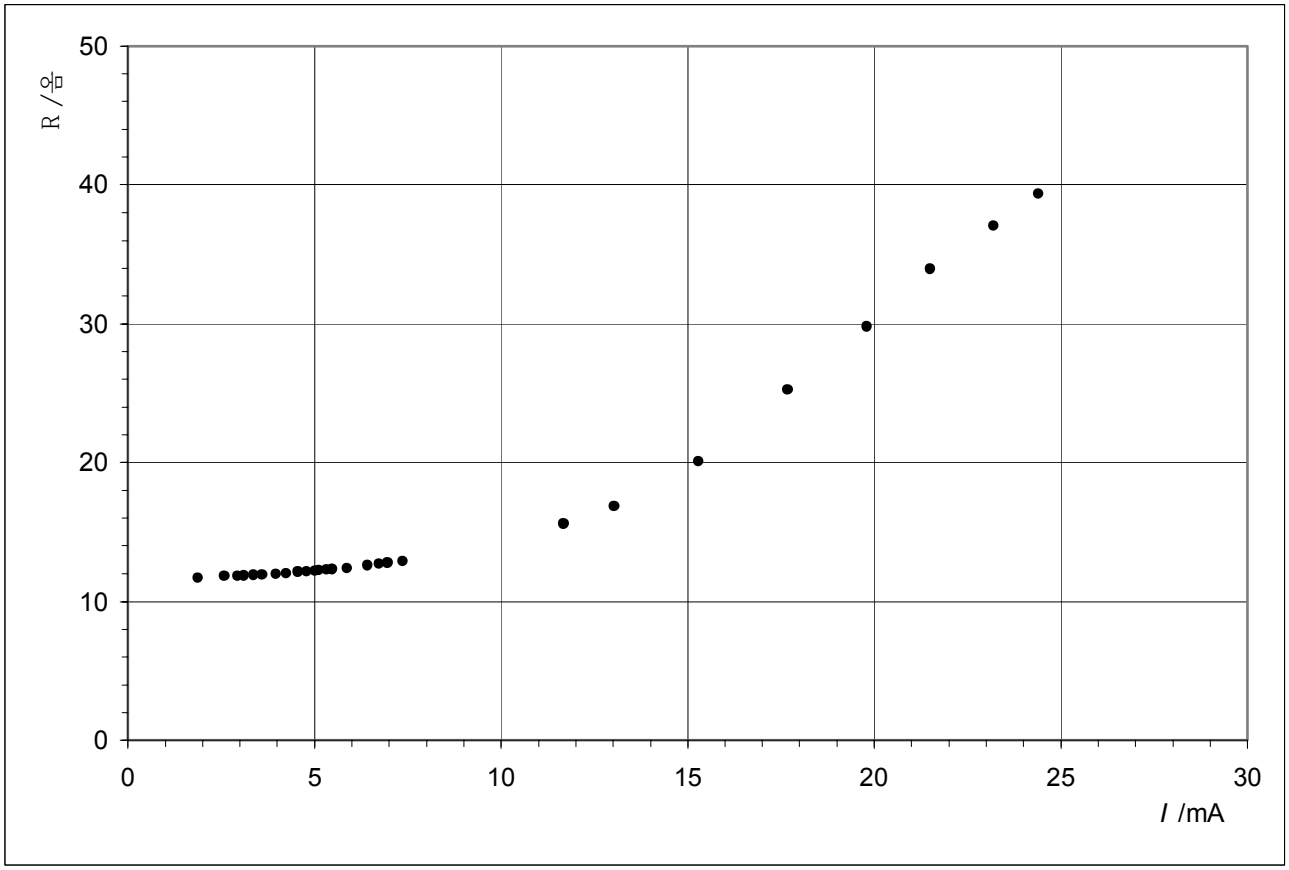
b)

V /mV	I / mA	R_B /Ω
21.9	1.87	11.7
30.5	2.58	11.8
34.9	2.95	11.8
37.0	3.12	11.9
40.1	3.37	11.9
43.0	3.60	11.9
47.6	3.97	12.0
51.1	4.24	12.1
55.3	4.56	12.1
58.3	4.79	12.2
61.3	5.02	12.2
65.5	5.33	12.3
67.5	5.47	12.3
73.0	5.88	12.4
80.9	6.42	12.6
85.6	6.73	12.7
89.0	6.96	12.8
95.1	7.36	12.9
111.9	8.38	13.4
130.2	9.37	13.9
181.8	11.67	15.6
220	13.04	16.9
307	15.29	20.1
447	17.68	25.1
590	19.8	29.8
730	21.5	33.9
860	23.2	37.1
960	24.4	39.3

$V_{min} =$ 9.2 mV *

* 이것은 여러분 장치의 특성이다. 이보다 낮출 수 없다.

세로축에 R_B, 가로축에 I를 두고 나타낸다.

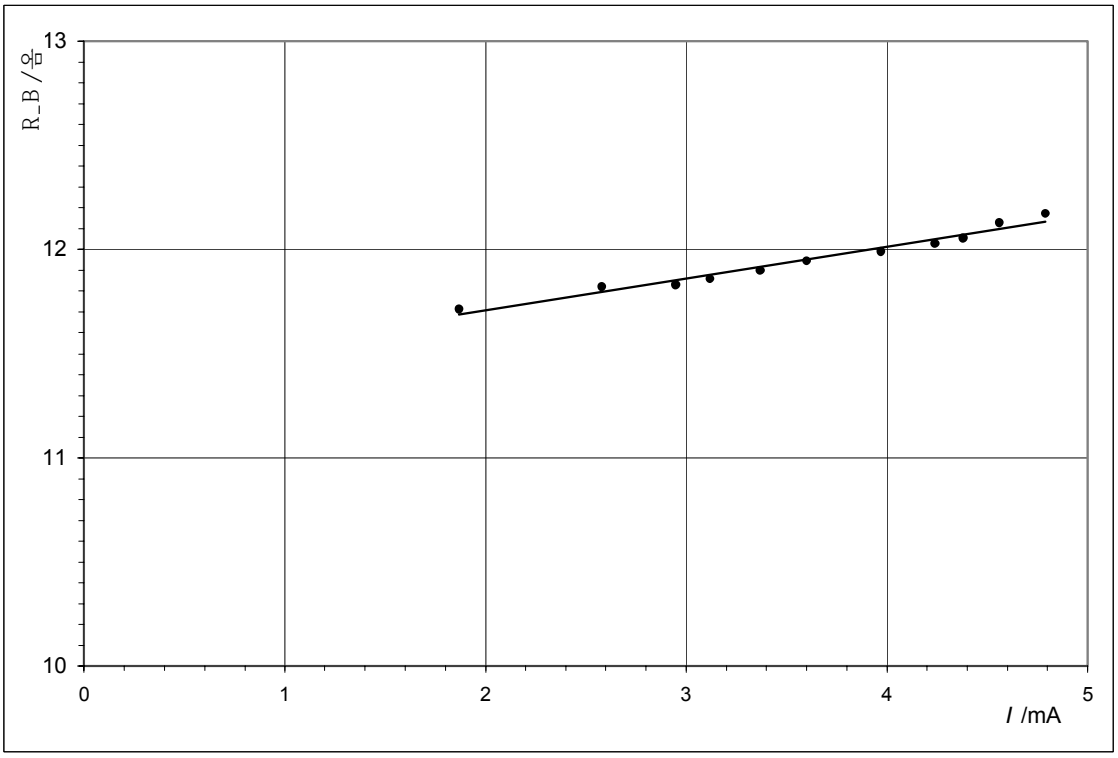


R_{B0}을 구하려고 처음 열 개의 읽음을 고른다.

과제 2

c)

V /mV	I / mA	R_B /Ω
21.9 ± 0.1	1.87 ± 0.01	11.7 ± 0.1
30.5 ± 0.1	2.58 ± 0.01	11.8 ± 0.1
34.9 ± 0.1	2.95 ± 0.01	11.8 ± 0.1
37.0 ± 0.1	3.12 ± 0.01	11.9 ± 0.1
40.1 ± 0.1	3.37 ± 0.01	11.9 ± 0.1
43.0 ± 0.1	3.60 ± 0.01	11.9 ± 0.1
47.6 ± 0.1	3.97 ± 0.01	12.0 ± 0.1
51.1 ± 0.1	4.24 ± 0.01	12.1 ± 0.1
55.3 ± 0.1	4.56 ± 0.01	12.1 ± 0.1
58.3 ± 0.1	4.79 ± 0.01	12.2 ± 0.1



R_B의 오차(예로 첫 번째 값의 오차를 구한다).

$$\Delta R_B = R_B\sqrt{\left(\frac{\Delta V}{V}\right)^2+\left(\frac{\Delta I}{I}\right)^2} = 11.71\sqrt{\left(\frac{0.1}{21.9}\right)^2+\left(\frac{0.01}{1.87}\right)^2} = 0.1$$

R_{B0}은 최소제곱법으로 구했다.

$R_{B0} = 11.4$

기울기 $m = 0.167$

$\sum I^2 = 130.38$

$\sum I = 35.05$

$n = 10$

X축: $X : \sigma_I = \sqrt{\frac{\sum \Delta I^2}{n}} = 0.01$

Y축: $Y : \sigma_{R_B} = \sqrt{\frac{\sum \Delta R_B^2}{n}} = 0.047$

$$\sigma = \sqrt{\sigma_{R_B}^2 + m^2\sigma_I^2} = \sqrt{0.1^2 + 0.167^2 \cdot 0.01^2} = 0.1$$

$$\Delta R_{B0} = \sqrt{\frac{\sigma^2\sum I^2}{n\sum I^2 - \left(\sum I\right)^2}} = \sqrt{\frac{0.1^2 \times 130.38}{10 \cdot 130.38 - 35.05^2}} = 0.13$$

$R_{B0} = 11{,}4\ \Omega$	$\Delta R_{B0} = 0.1\ \Omega$

d) $T = aR^{0.83}$; $a = \frac{T_0}{R_0^{0.83}}$; $a = \frac{297}{11.4^{0.83}} = 39.40$

두 방법으로 오차를 구한다:

방법 A

$\ln a = \ln T_0 - 0.83 \ln R_{B0}$; $\Delta a = a\left(\frac{\Delta T_0}{T_0} + 0.83\frac{\Delta R_{B0}}{R_{B0}}\right)$; $\Delta a = 39.40\left(\frac{1}{297} + 0.83\frac{0.1}{11.40}\right) = 0.419 = 0.4$

방법 B

a의 큰 값: $a_{\max} = \frac{T_0 + \Delta T_0}{(R_0 - \Delta R_0)^{0.83}} = \frac{297 + 1}{(11.4 - 0.1)^{0.83}} = 39.8255$

a의 작은 값: $a_{\min} = \frac{T_0 - \Delta T_0}{(R_0 + \Delta R_0)^{0.83}} = \frac{297 - 1}{(11.4 + 0.1)^{0.83}} = 38.9863$

$\Delta a = \frac{a_{\max} - a_{\min}}{2} = \frac{39.8255 - 38.9863}{2} = 0.419 = 0.4$

$a = 39.4$	$\Delta a = 0.4$

과제 3

다음 까닭으로 $2\Delta\lambda = 620 - 565$; $\Delta\lambda = 28$ nm

$\lambda_0 = 590$ nm	$\Delta\lambda = 28$ nm

과제 4

a)

V /V	I / mA	R /kΩ
9.48	85.5	8.77
9.73	86.8	8.11
9.83	87.3	7.90
100.1	88.2	7.49
10.25	89.4	7.00
10.41	90.2	6.67
10.61	91.2	6.35
10.72	91.8	6.16
10.82	92.2	6.01
10.97	93.0	5.77
11.03	93.3	5.69
11.27	94.5	5.35
11.42	95.1	5.17
11.50	95.5	5.07

b)

다음 까닭으로 $\ln\frac{R}{R'} = \gamma \ln 0.512$; $\gamma = \ln\frac{R}{R'} \Big/ \ln 0.51\,2 = \ln\frac{5.07}{8.11} \Big/ \ln 0.512 = 0.702$

△γ를 구하려면 다음을 안다:

$R \pm \Delta R = 5.07 \pm 0.01\ \text{k}\Omega$

$R' \pm \Delta R' = 8.11 \pm 0.01\ \text{k}\Omega$

투과율, $t = 51.2\ \%$

두 방법으로 오차를 구한다:

방법 A

$$\gamma = \frac{\ln R/R'}{\ln t}\ ;\ \Delta\gamma = \frac{1}{\ln t}\left(\frac{\Delta R}{R} + \frac{\Delta R'}{R'}\right) = \frac{1}{\ln 0.512}\left(\frac{0.01}{5.07} + \frac{0.01}{8.11}\right) = 0.00479\ ;\ \Delta\gamma = 0.005$$

방법 B

γ의 큰 값: $\gamma_{\max} = \ln\frac{R - \Delta R}{R'+\Delta R'} \Big/ \ln\gamma = \ln\frac{5.07 - 0.01}{8.11 + 0.01} \Big/ \ln 0.512 = 0.70654$

γ의 작은 값: $\gamma_{\max} = \ln\frac{R + \Delta R}{R'-\Delta R'} \Big/ \ln\gamma = \ln\frac{5.07 + 0.01}{8.11 - 0.01} \Big/ \ln 0.512 = 0.69696$

$$\Delta\gamma = \frac{\gamma_{\max} - \gamma_{\min}}{2} = \frac{0.70654 - 0.69696}{2} = 0.00479; \quad \Delta\gamma = 0.005$$

$R = 5.07\ \text{k}\Omega$	$\gamma = 0.702$
$R' = 8.11\ \text{k}\Omega$	$\Delta\gamma = 0.005$

c)

다음을 안다 $R = c_3 e^{\frac{c_2\gamma}{\lambda_0 T}}$ (3)

그러면 $\ln R = \ln c_3 + \frac{c_2\gamma}{\lambda_0 T}$

까닭은 $T = aR_B^{0.83}$ (6)

따라서 $\ln R = \ln c_3 + \frac{c_2\gamma}{\lambda_0 a} R_B^{-0.83}$

$$\ln R = \ln c_3 + \frac{c_2\gamma}{\lambda_0 a} R_B^{-0.83} \quad \text{Eq.(9)}$$

d)

V /V	I / mA	R_B / Ω	T / K	$R_B^{-0.83}$ (S.I.)	R / kΩ	ln R
9.48 ± 0.01	85.5 ± 0.1	110.9 ± 0.2	1962 ± 18	$(2.008 \pm 0.004)10^{-2}$	8.77 ± 0.01	2.171 ± 0.001
9.73± 0.01	86.8 ± 0.1	112.1 ± 0.2	1980 ± 18	$(1.990\pm 0.004)10^{-2}$	8.11 ± 0.01	2.093 ± 0.001
9.83± 0.01	87.3 ± 0.1	112.6 ± 0.2	1987 ± 18	$(1.983\pm 0.004)10^{-2}$	7.90 ± 0.01	2.067 ± 0.001
10.01± 0.01	88.2 ± 0.1	113.5 ± 0.2	2000 ± 18	$(1.970\pm 0.004)10^{-2}$	7.49 ± 0.01	2.014 ± 0.001
10.25± 0.01	89.4 ± 0.1	114.7 ± 0.2	2018 ± 18	$(1.952\pm 0.003)10^{-2}$	7.00 ± 0.01	1.946 ± 0.001
10.41± 0.01	90.2 ± 0.1	115.4 ± 0.2	2028 ± 18	$(1.943\pm 0.003)10^{-2}$	6.67 ± 0.01	1.894 ± 0.002
10.61± 0.01	91.2 ± 0.1	116.3 ± 0.2	2041 ± 18	$(1.930\pm 0.003)10^{-2}$	6.35 ± 0.01	1.849 ± 0.002
10.72± 0.01	91.8 ± 0.1	116.8 ± 0.2	2049 ± 19	$(1.923\pm 0.003)10^{-2}$	6.16 ± 0.01	1.818 ± 0.002
10.82± 0.01	92.2 ± 0.1	117.4 ± 0.2	2057 ± 19	$(1.915\pm 0.003)10^{-2}$	6.01 ± 0.01	1.793 ± 0.002
10.97± 0.01	93.0 ± 0.1	118.0 ± 0.2	2066 ± 19	$(1.907\pm 0.003)10^{-2}$	5.77 ± 0.01	1.753 ± 0.002
11.03± 0.01	93.3 ± 0.1	118.2 ± 0.2	2069 ± 19	$(1.904\pm 0.003)10^{-2}$	5.69 ± 0.01	1.739 ± 0.002
11.27± 0.01	94.5 ± 0.1	119.3 ± 0.2	2085 ± 19	$(1.890\pm 0.003)10^{-2}$	5.35 ± 0.01	1.677 ± 0.002
11.42± 0.01	95.1 ± 0.1	120.1 ± 0.2	2096 ± 19	$(1.880\pm 0.003)10^{-2}$	5.15 ± 0.01	1.639 ± 0.002
11.50± 0.01	95.5 ± 0.1	120.4 ± 0.2	2101 ± 19	$(1.875\pm 0.003)10^{-2}$	5.07 ± 0.01	1.623 ± 0.002

필요 없음

예로 첫째 줄 전체의 오차를 구한다.

R_B의 오차: $\Delta R_B = R_B\sqrt{\left(\frac{\Delta V}{V}\right)^2+\left(\frac{\Delta I}{I}\right)^2} = 110.9\sqrt{\left(\frac{0.01}{9.48}\right)^2+\left(\frac{0.1}{85.5}\right)^2} = 0.2\ \Omega$

T의 오차: $\Delta T = T\left(\frac{\Delta a}{a}+0.83\frac{\Delta R_B}{R_B}\right)$; $\Delta T = 1962\left(\frac{0.3}{39.4}+0.83\frac{0.2}{110.9}\right) = 18\,\mathrm{K}$

R_B의 오차 $^{-0.83}$:

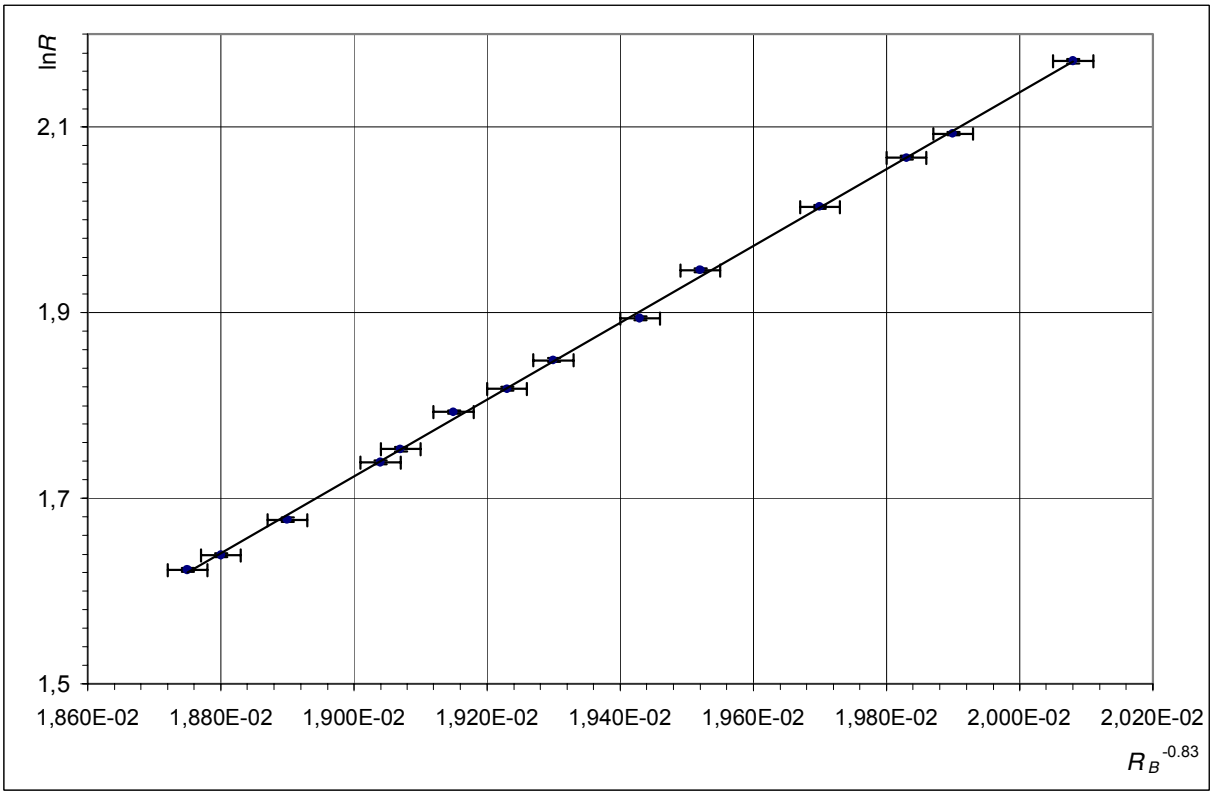
$$x = R_B^{-0.83}\ ;\ \ln x = -0.83\ln R_B\ ;\ \Delta x = x\cdot 0.83\frac{\Delta R_B}{R_B}\ ;\ \Delta\left(R_B^{-0.83}\right) = R_B^{-0.83}\frac{\Delta R_B}{R_B}$$

$$\Delta\left(R_B^{-0.83}\right) = 0.020077\frac{0.2}{110.9} \approx 0.004\times 10^{-2}$$

ln R의 오차: $\Delta \ln R = \frac{\Delta R}{R}$; $\Delta \ln R = \frac{0.01}{8.77} = 0.001$

e)

$R_B^{-0.83}$에 대해 ln R을 그린다.



최소제곱법으로

기울기 = $m = 414{,}6717$

$$\sum \left(R_B^{-0.83}\right)^2 = 5.23559 \times 10^{-3}$$

$$\sum \left(R_B^{-0.83}\right) = 0.27068$$

$$n = 14$$

X축: $$X: \sigma_{R_B^{-0.83}} = \sqrt{\frac{\sum \Delta\left(R_B^{-0.83}\right)^2}{n}} = 0.003 \times 10^{-2}$$

Y축: $$Y: \sigma_{\ln R} = \sqrt{\frac{\sum \Delta(\ln R)^2}{n}} = 0.002$$

$$\sigma = \sqrt{\sigma_{\ln R}^2 + m^2 \sigma_{R_B^{-0.83}}^2} = \sqrt{0.002^2 + 414.672^2 \cdot \left(0.003 \times 10^{-2}\right)^2} = 0.0126$$

$$\Delta m = \sqrt{\frac{n\sigma^2}{n\sum \left(R_B^{-0.83}\right)^2 - \left(\sum R_B^{-0.83}\right)^2}} = \sqrt{\frac{14 \cdot 0{,}0126^2}{14 \cdot 5.23559 \times 10^{-3} - (0.27068)^2}} = 8.295$$

다음 까닭으로

$$m = \frac{c_2 \gamma}{\lambda_0 a}$$

그리고

$$c_2 = \frac{hc}{k}$$

그러면

$$h = \frac{mk\lambda_0 a}{c\gamma}$$

$$h = \frac{414.67 \cdot 1.381 \times 10^{-23} \cdot 590 \times 10^{-9} \cdot 39.4}{2.998 \times 10^{8} \cdot 0.702} = 6.33 \times 10^{-34}$$

$$\Delta h = h\sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta k}{k}\right)^2 + \left(\frac{\Delta \lambda_0}{\lambda_0}\right)^2 + \left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta \gamma}{\gamma}\right)^2}$$

$$\Delta h = 6.34 \times 10^{-34}\sqrt{\left(\frac{8.3}{415}\right)^2 + 0 + \left(\frac{28}{590}\right)^2 + \left(\frac{0.3}{39.4}\right)^2 + 0 + \left(\frac{0.01}{0.70}\right)^2} = 0.34 \times 10^{-34}$$

$h = 6.3 \times 10^{-34}\ \mathrm{J \cdot s}$	$\Delta h = 0.3 \times 10^{-34}\ \mathrm{J \cdot s}$