EXPERIMENTAL EXAMINATION

Experimental problem

Determine a ball coefficient of restitution when it bounces on the floor.

Theory. In a collision between a ball and Earth, the mass of Earth may be treated as infinite. In an inelastic collision kinetic energy is not conserved. Let *V* be the ball velocity on reaching the floor and *v* its velocity just after rebounding, so that:

$$\frac{1}{2}mV^2 \geq \frac{1}{2}mv^2$$

Equality corresponds to an elastic collision.
Hence

$$V^2 \geq v^2 \quad \Rightarrow \quad \left(\frac{v}{V}\right)^2 = \varepsilon^2 \leq 1$$

The ratio of the speed after impact to that before impact is defined as the *coefficient of restitution* of the inelastic collision.

$$\varepsilon = \frac{v}{V} \leq 1$$

If the ball is released from height *H*, the time until it first reaches the floor is:

$$t_0 = \sqrt{\frac{2H}{g}}$$

Its velocity immediately before the first bounce is:

$$v_0 = gt_0$$

Its velocity immediately after the first bounce is:

$$v_1 = \varepsilon v_0$$

Let t_1 , t_2 , ... , t_{n-1} be the times from release to the second, third, ..., *n* bounces respectively.
The interval between the first and second bounces is

$$\Delta t_1 = t_1 - t_0 = \frac{2v_1}{g} = \varepsilon\frac{2v_0}{g} = 2t_0\varepsilon$$

The velocity immediately after the second bounce is

$$v_2 = \varepsilon v_1 = \varepsilon^2 v_0$$

The interval between the second and third bounces is

$$\Delta t_2 = t_2 - t_1 = \frac{2v_2}{g} = \varepsilon^2 \frac{2v_o}{g} = 2t_0\varepsilon^2$$

Continuing in the same way gives

$$\Delta t_n = t_n - t_{n-1} = 2t_0\varepsilon^n$$

Thus the interval between successive bounces is theoretically proportional to the *n*th power of the coefficient of restitution, where *n* counts the bounces after the first.

NOTE: Always release the ball from the greatest height your arm allows.
This ensures that the relative height error in each drop is smaller than the relative timing error, and can be neglected.