IX Iberoamerican Physics Olympiad

Salvador, September 2004

1 Experiment I: Viscosity measurements and Stokes law

1.1 Aim

Study the viscous drag exerted by a fluid (automotive lubricating oil) on objects moving within it.

1.2 Introduction

Fluid mechanics studies fluid motion in pipes and solid motion through fluids. Generally, relative motion of solids and fluids is described simply only at low speeds. This regime is called laminar flow: the fluid behaves as thin layers sliding over one another, with friction in real fluids. Their shape depends on the problem symmetry. For laminar flow we define the dynamic viscosity η.

G. G. Stokes showed in 1850 that in laminar flow the drag on a sphere of radius r moving at velocity $\vec{v}$ through an infinite medium of viscosity η is:

$$\vec{F}_S = -6\pi\eta r\vec{v}. \tag{1}$$

Nearby walls require corrections to Stokes law (1). For a sphere falling along the axis of a cylinder of length H and inner radius $R > r$, R. Ladenburg, O. Emersleben and H. Faxen found the corrected drag $\vec{F}_S = -6\pi\eta\lambda r\vec{v}$,

where at low speeds, for $R \gg r$ and $H \gg r$,

$$\lambda = \left(\frac{1 + 3,3\left(\frac{r}{H}\right)}{1 - 2,104\left(\frac{r}{R}\right) + 2,09\left(\frac{r}{R}\right)^3} \right) .$$

1.3 Question

Consider the sphere falling vertically along the cylinder axis. Use Newton second law to write the equation of motion under weight, drag and buoyancy. Viscous drag rapidly brings the speed near a terminal value v_{lim}. Show that it is given by

$$v_{\text{lim}} = \frac{2(\rho_{\text{esf}} - \rho)}{9\eta\lambda} g r^2 ,$$

where ρ_{esf} is the sphere density and g the gravitational acceleration.

1.4 Experiment

1.4.1 Material

A cylinder with about 1 litre of automotive oil, 20 metal spheres, diameters
1,5 mm, 2,0 mm, 2,5 mm and 3,0 mm (5 each), a stopwatch, ruler, adhesive tape and graph paper. Oil density: 0,89 g/cm^3. Sphere density: $7,8 \pm 0,1$ g/cm^3. Cylinder inner diameter: 60 $,5 \pm 0,5$ mm. Take $g = 9,8$ m/s^2.

1.4.2 Observed data and Analysis

1. Measure terminal falling speeds for different sphere diameters, releasing each as near the cylinder axis as possible. Describe your procedure in detail and estimate measurement errors.

2. On graph paper, study v_{lim} as a function of sphere radius r.

3. Determine the oil viscosity η and estimate its measurement error.

4. The flow regime can be assessed using the dimensionless Reynolds number. Here it may be defined as

$$\mathcal{R} = \frac{2r\rho v_{\text{lim}}}{\eta}, \tag{2}$$

where ρ is fluid density. $\mathcal{R} \ll 1$ indicates that laminar flow is a reasonable approximation. Check this condition for the experiments with the different spheres.