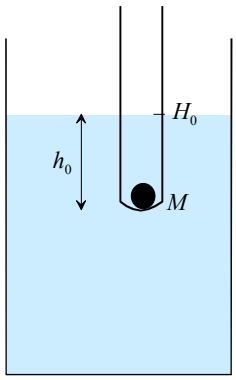
EXPERIMENT 1: Spirit hydrometer

WORKED EXAMPLE:

Experimental arrangement:



Make the following considerations initially or during the experiment:

M = tube mass + added mass needed to keep it vertical.
$V_0 = h_0\ \pi\ r^2$ is the volume immersed at equilibrium with mass M; r is the tube outer radius and h_0 its immersed length.

At equilibrium, the total weight equals the buoyant force:

$$P = I$$

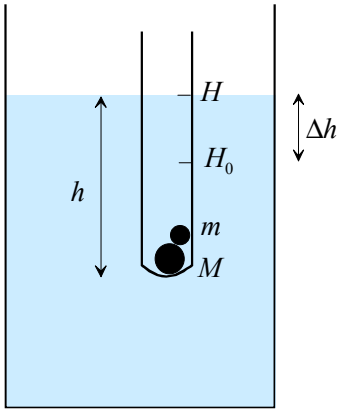
or

$$M\,g = V_0\,\rho_L\,g$$

where ρ_L is the liquid density and g gravitational acceleration. Thus,

$$M = V_0\,\rho_L$$

Adding a mass m makes the tube sink to a new equilibrium position.



At the new equilibrium position:

$$(M + m)\, g = V\, \rho_L\, g$$
$$M + m = V_0\, \rho_L + \Delta V\, \rho_L$$
$$m = \Delta V\, \rho_L$$

Since $\Delta V = \pi\, r^2\, \Delta h$, we obtain

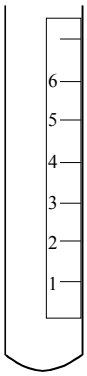
$$m = \pi\, r^2\, \Delta h\, \rho_L$$

$$\boxed{\Delta h = \frac{1}{\rho_L\, \pi\, r^2}\, m}$$

The proportionality constant between immersion depth and added mass depends on the liquid density and the tube radius.

A) STATIC METHOD

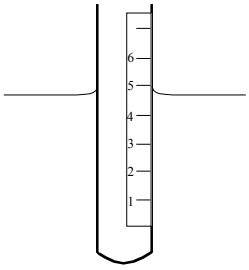
1 - To read the immersion depth, a piece of graph paper with a marked scale was placed inside the tube:



Tube mass including graph paper: M_{tubo} = 78,8 g ± 0,1g Number of lead pellets added = 80 Added lead mass: 80 × (0,88 g ± 0,01 g) = 70,4 g ± 0,8 g Total tube-and-lead mass:

M = (78,8 g ± 0,1 g) + (70,4 g ± 0,8 g) = 149,2 g ± 1,7 g
M = 0,1492 kg ± 0,0017 kg

Test-tube diameter (calliper): d = 29,70 mm ± 0,05 mm Tube radius: r = 14,85 mm ± 0,03 mm r = 0,01485 m ± 0,00003 m Immersion depth with mass M: h_0 = 11,9 cm ± 0,1 cm. Always read h at the top of the meniscus.



2 - To find immersion depth versus added mass, 10 pellets were added at a time and the equilibrium depth was recorded. The results are tabulated below.

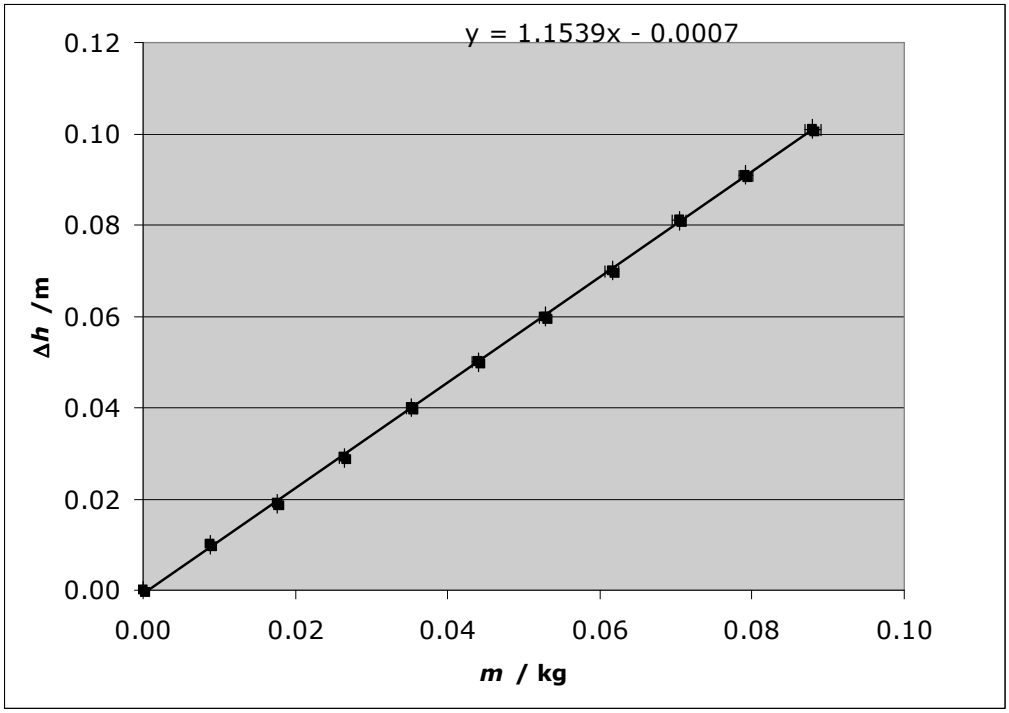
Lead pellets added	**Depth *h* /cm (± 0,1 cm)**
0	11,9
10	10,9
20	10,0
30	9,0
40	7,9
50	6,9
60	5,9
70	4,9
80	3,8
90	2,8
100	1,8

The next table gives these values, the total added mass m, and height change Δh:

Pellet count	***m* ×10^3 / kg**	***h* / m (± 0,001 m)**	**Δ*h* / m (± 0,002 m)**
0	0	0,119	0
10	8,80 ± 0,03	0,109	0,010
20	17,60 ± 0,05	0,100	0,019
30	26,40 ± 0,05	0,090	0,029
40	35,20 ± 0,06	0,079	0,040
50	44,00 ± 0,07	0,069	0,050
60	52,80 ± 0,08	0,059	0,060
70	61,60 ± 0,08	0,049	0,070
80	70,40 ± 0,09	0,038	0,081
90	79,20 ± 0,09	0,028	0,091
100	88,0 ± 0,1	0,018	0,101

3 - The next figure plots the measurements and their errors. The mass error (Δm) is small and lies within the plotted

point symbol. The equation of the best-fit straight line is shown.



The slope is m = 1,154 m kg^{-1}. Hence,

$$1{,}154 = \frac{1}{\rho_L \pi r^2}$$

the liquid density is ρ_L =1/(1,154 π r^2). Substituting known values:

$$\rho_L = 1251 \text{ kg m}^{-3}$$

4 - Since the regression gives no slope error, estimate it from two graph points, (x,y) = (0,0352; 0,040) and (0,088; 0,101):

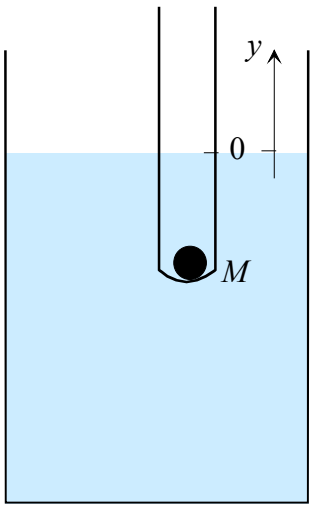
$$m = \frac{(0{,}101 \pm 0{,}002) - (0{,}041 \pm 0{,}002)}{(0{,}088 \pm 0{,}002) - (0{,}035 \pm 0{,}002)} = 1{,}15 \pm 0.06$$

With this error, recalculate the liquid density:

ρ_L =1/[1,154±0.06 × 3.1416 × (0,01485^2)±4x10^{-5}] = (**1251 ± 211) kg m^{-3}**

B) DYNAMIC METHOD

5 - For a fixed tube-and-lead mass, a vertical displacement from equilibrium changes the buoyancy and gives a restoring force $F = -\Delta V \rho_L g$.



The preceding minus sign ensures that on sinking $\Delta V < 0$ ($y < 0$) and $F > 0$, with the reverse for $y > 0$. Substituting ΔV in terms of the tube radius and y gives:

$$F = -(\pi r^2 y)\,\rho_L g$$
$$F = -(\pi r^2 \rho_L g)\,y$$

This is an elastic force $F = -k\,x$, with

$$k = \pi r^2 \rho_L g$$

and the angular frequency is $w = \dfrac{2\pi}{T} = \sqrt{\dfrac{k}{M}}$, where M is the total oscillating mass of tube and lead. The period is:

$$T = 2\pi\sqrt{\frac{M}{\pi r^2 \rho_L g}}$$

Here r, g and M are known, so measuring T gives ρ_L:

$$\rho_L = \frac{4\pi M}{T^2 r^2 g}$$

6 - To keep the tube vertical, 100 lead pellets were added.

Total mass of tube and lead pellets: M = 166,8 g ± 0,1 g

To measure T, push the tube down vertically and release it, starting the stopwatch. Record the time for 5 oscillations. The results are tabulated below:

Oscillations	time (/s)	T (/s)
5	4,72	0,944
5	4,62	0,924
5	4,69	0,938
5	4,53	0,906
5	4,53	0,906
5	4,69	0,938
5	4,53	0,906
5	4,61	0,922
5	4,52	0,904

The mean and standard deviation give:

$$T = 0{,}92 \text{ s} \pm 0{,}02 \text{ s}$$

This error includes only statistical variation; systematic errors may also be present.

7 -

Using the density relation with period T, $\rho_{\mathrm{L}} = \dfrac{4\pi M}{T^2 r^2 g}$,, and substituting known values gives

$$\rho_{\mathrm{L}} = \frac{4 \times \pi (0{,}1668 \pm 0{,}0001)}{(0{,}92 \pm 0{,}02)^2 \times (0{,}01485 \pm 0{,}00003)^2 \times (9{,}81 \pm 0{,}01)}$$

$$\boxed{\rho_{\mathrm{L}} = (1143 \pm 185) \text{ kg m}^{-3}.}$$

The two liquid-density values

A) 1251 kg m^{-3} ± 211 kg m^{-3} and **B) 1143 kg m^{-3} ± 185 kg m^{-3},**

agree within experimental error.

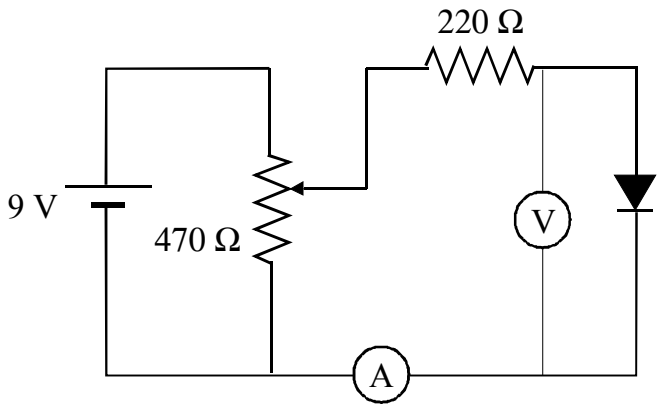
The second method neglects water viscosity. This has two effects: (1) the motion is damped rather than simple harmonic, as observed; (2) some water also oscillates, adding an effective mass to M. Since the measured M is used rather than this larger corrected mass, the density found by this method should be below the true value.

EXPERIMENT 2: Light-emitting diodes (LEDs) and Planck constant

Worked example

Measure the resistors to identify the 220 Ω and 11,4 kΩ resistors.

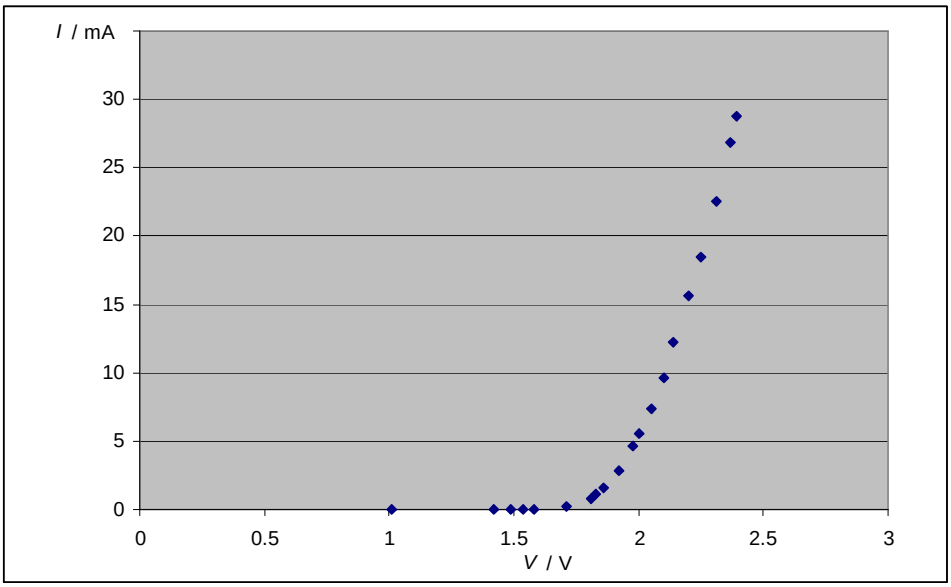
A) 1 - Assemble the circuit below, using the potentiometer as a voltage divider:



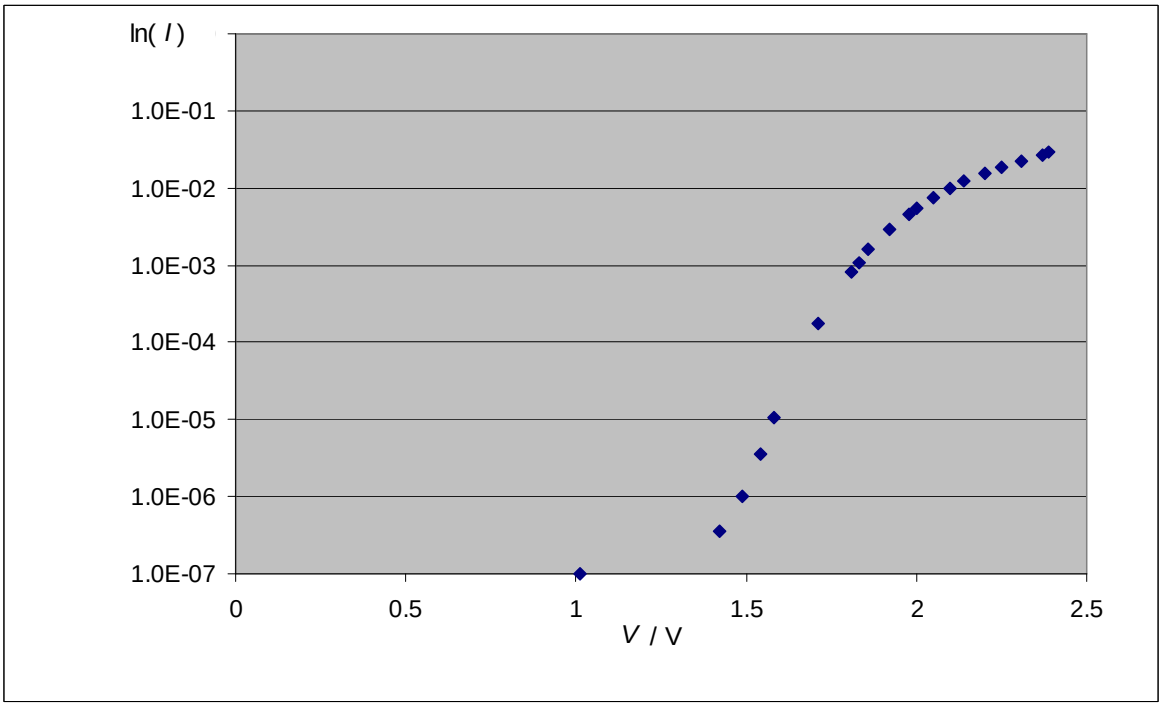
2 - Characteristic curve of the red LED:
With this circuit, vary the red LED voltage using the potentiometer and record the current *I* and terminal voltage *V*. The logarithm of the current in amperes is also listed:

V **/ V**	***I*** **/ mA**	**ln *I***
1,01	0,00010	-16,118
1,42	0,00035	-14,865
1,49	0,00102	-13,796
1,54	0,00365	-12,521
1,58	0,01065	-11,450
1,71	0,17180	-8,669
1,81	0,82400	-7,101
1,83	1,09500	-6,817
1,86	1,63700	-6,415
1,92	2,85000	-5,860
1,98	4,64000	-5,373
2,00	5,50000	-5,203
2,05	7,37000	-4,910
2,10	9,67000	-4,639
2,14	12,23000	-4,404
2,20	15,58000	-4,162
2,25	18,42000	-3,994
2,31	22,50000	-3,794
2,37	26,90000	-3,616
2,39	28,80000	-3,547

The graph shows these values:

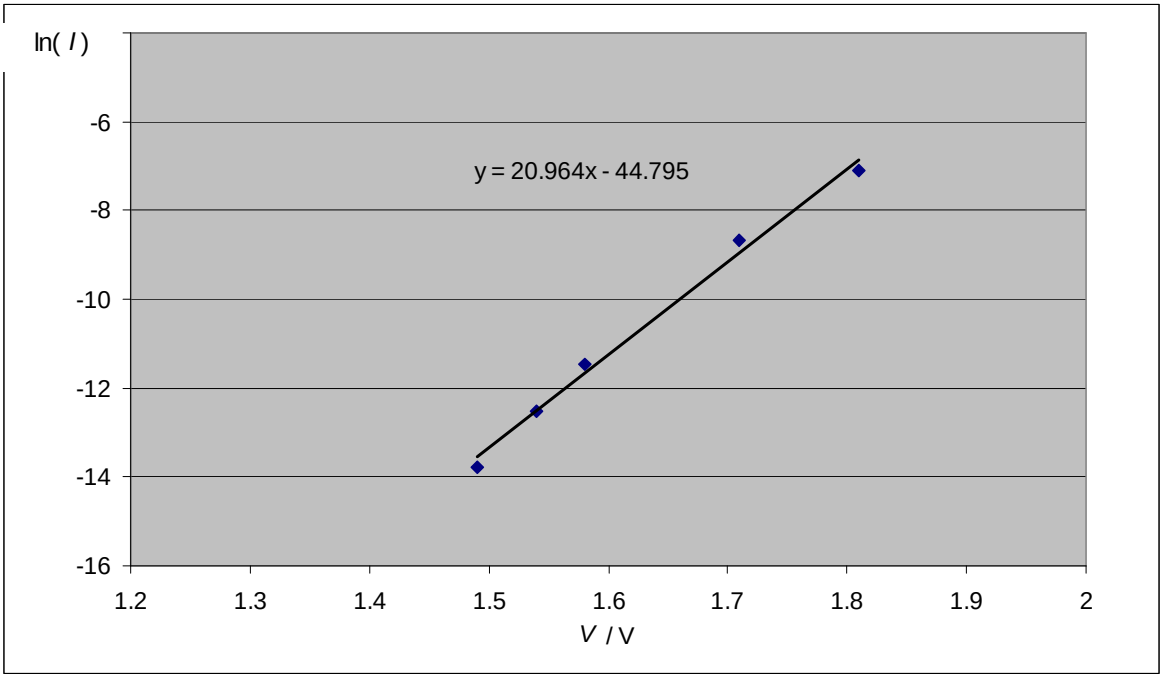


The characteristic curve can also be plotted on semi-logarithmic axes:



The vertical scale is logarithmic, so the expression predicts a straight line above a certain voltage. This occurs from about 1,5 to 1,8 V. Above ~1,8V the slope changes as the diode enters a different, resistive regime where the expression is no longer valid.

3 - Find η and I_0 using points in the linear region, where the theoretical expression holds (marked in the table). These points give:



The best-fit line ($y = m\ x + b$) gives η and I_0.
From

$$I = I_0\ \mathrm{e}^{\frac{eV}{\eta\, k_\mathrm{B} T}}$$

taking the logarithm

$$\ln(I) = \ln(I_0) + \frac{e}{\eta\, k_\mathrm{B} T} V$$

gives

$$b = \ln(I_0) \qquad m = \frac{e}{\eta\, k_\mathrm{B}\, T}$$

hence:

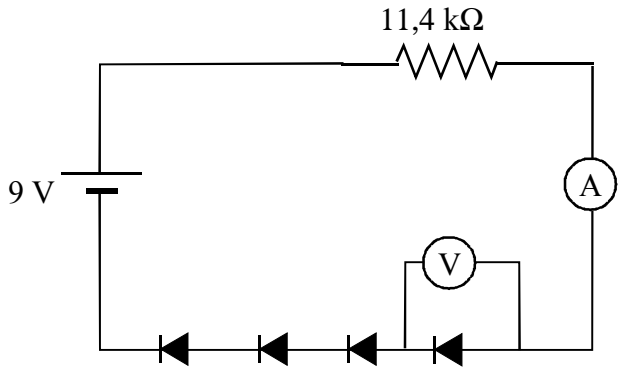
$$I_0 = \mathrm{e}^{b}$$

$$\eta = \frac{e}{m\, k_\mathrm{B}\, T}$$

Thus, using the fitted slope and intercept,

I_0 $= 3{,}5 \times 10^{-20}$ A | $\eta =$ 1,9

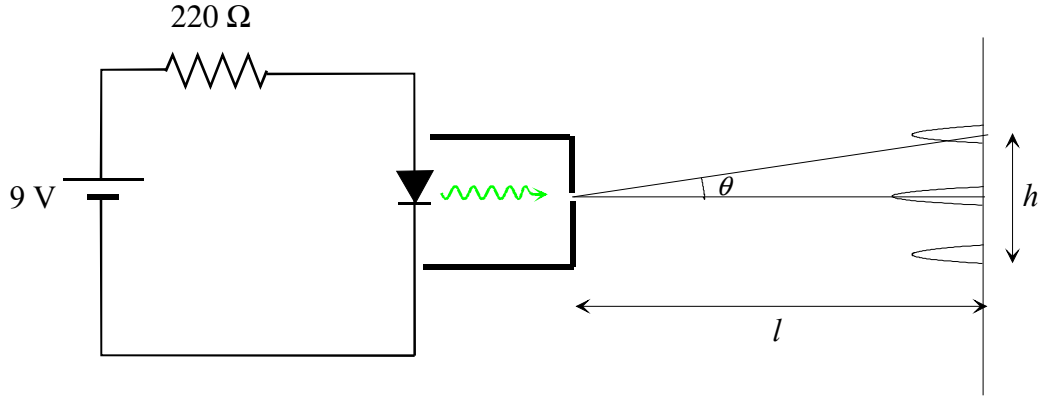
4 - Series connection of the LEDs:



Each LED carries I = 118 μA. The table lists each terminal voltage, wavelength and emitted frequency:

LED	λ / nm	ν / Hz	*V* /V
Infrared	950	3,16E+14	1,178
Red	630	4,76E+14	1,755
Green	??	??	2,359
Blue	470	6,38E+14	2,475

B) Wavelength of the green LED:
5) Connect the green LED to the battery in series with the 220 Ω resistor.



Place the diffraction grating with the black tube as shown, and an A3-sheet screen a distance l away. Denote the separation of the first diffraction maxima on either side of the central maximum by h.

Results: l = 0,278 m ± 0,002 m h = 0,350 m ± 0,005 m d = 1/1000 mm = 1×10^{-6} m (given)

The value of θ is:

$$\theta = \arctan\left(\frac{h}{2l}\right), \qquad \theta = 0{,}56 \pm 0{,}02 \text{ rad}$$

Using the expression (n = 1)

$$d \sin\theta = \lambda$$

we obtain

$\lambda = 533 \text{ nm} \pm 20 \text{ nm}$

assuming negligible uncertainty in the grating spacing d.

C)
6 -

Green LED frequency: $\nu = \frac{c}{\lambda} = 5{,}62 \times 10^{14}$ Hz . Since the current is the same through all 3 LEDs, so

$$W = E_f + k$$

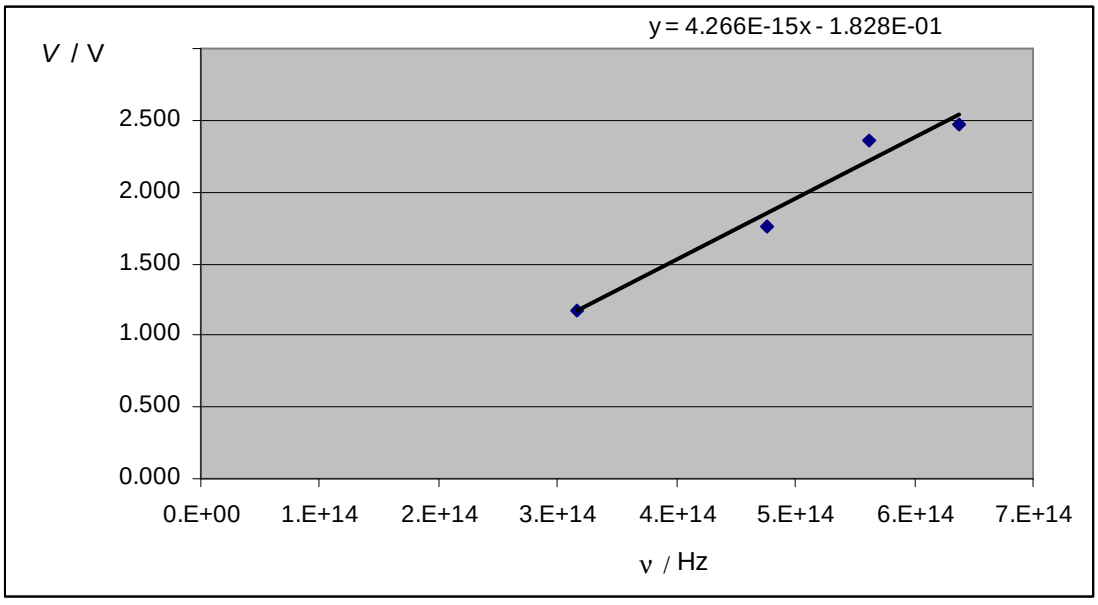
which gives

$$eV = h\nu + k$$

or equivalently

$$V = \frac{h}{e}\nu + k$$

The graph below plots the measured V against frequency ν for the four LEDs.



The slope m of the best-fit line gives:

$$h = m\,e = (6{,}83 \pm 0{,}89) \times 10^{-34}\ \text{J s}$$

The result is about 3% above the tabulated value, but the deviation is within experimental uncertainty.