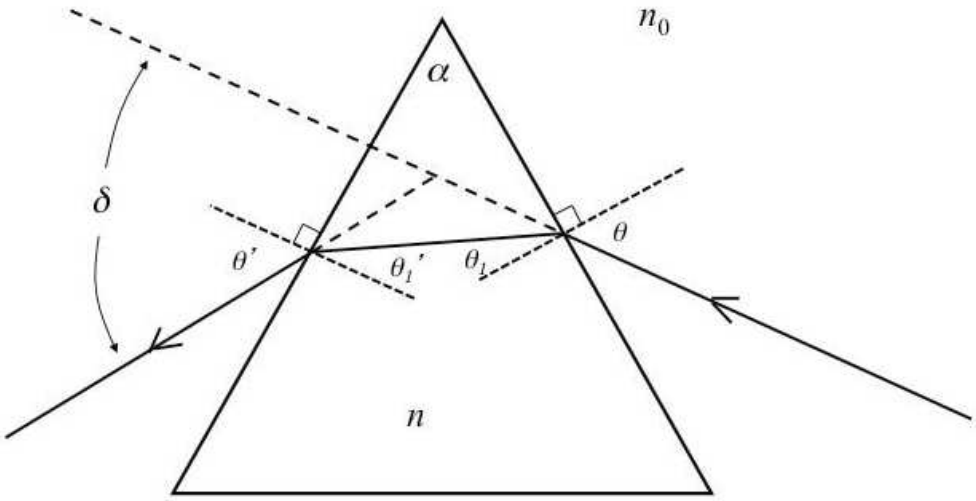
Experimental Problem

XIII Iberoamerican Physics Olympiad, 2008, Morelia, Michoacán, Mexico

Refractive index

The aim is to measure the refractive index n of a transparent material, and how the emergent beam angle depends on the incident beam angle. The material is a triangular prism made of resin. The physics is simple; see the figure.



A laser beam strikes one face of the prism at an angle θ to the normal. It is refracted into the prism at an angle θ_1, then strikes the adjacent face at an angle θ_1' and is finally refracted out at an angle θ'. Let α be the prism's apex angle. Let δ be the angle between the emergent beam and the incident beam direction. Take the refractive index of air as $n_0 = 1.0$.

Recall that the incidence and refraction angles of monochromatic light passing between media of refractive indices n_1 and n_2 are related by Snell's law:

$$n_1 \operatorname{sen}\varphi_1 = n_2 \operatorname{sen}\varphi_2 \qquad (1)$$

where φ_1 and φ_2 are the angles the beam makes with the normal to the interface in the respective media.

Snell's law and the geometry of the prism and beam
give a function relating the angle
δ to θ, namely $\delta = \delta(\theta)$. This function has
a minimum at angle θ_c. Denote this value by δ_{min}, so $\delta_{\text{min}} = \delta(\theta_c)$. It
can be shown that evaluating $\delta(\theta)$ at θ_c gives the following relation for δ_{min}:

$$n \operatorname{sen} \frac{\alpha}{2} = n_0 \operatorname{sen}\left(\frac{\delta_{\text{min}} + \alpha}{2} \right) \qquad (2)$$

WARNING: NEVER LOOK DIRECTLY INTO THE LASER BEAM!

EQUIPMENT (Use only what you need; not all items are necessary.)

- Prism
- Laser
- Wooden base
- Long wooden screen
- Small cardboard screen (10 cm x 10 cm)
- Graduated ruler
- Set squares
- White paper
- Millimetre graph paper
- Scissors
- Adhesive tape
- Pencils
- Sharpener
- Eraser
- Calculator
- Thread

Because the prism may have imperfections, ensure that the beam always enters through the same face.

Keep the incident beam at the middle of a face of the prism.

Always determine angles using the centre of the light spot. This keeps the uncertainty in the angle measurement and makes the beam width contribute only a systematic error.

TASK 1. Draw an experimental arrangement that lets you determine the angles δ and θ. (3 points)

TASK 2. Determine the prism's refractive index *n* using the minimum-deviation formula, equation (2). (8 points total)

(2.a) Determine the prism angle α . (1 point)
(2.b) Measure the minimum angle experimentally. Report your data in a table. (4 points)
(2.c) Calculate the uncertainties (errors) of angle α and the minimum angle δ_{min}. (1.5 points)
(2.d) Obtain the refractive index *n* and its uncertainty. (1.5 points)

TASK 3. Experimentally determine the dependence $\delta(\theta)$. (6 points total)

(3.a) Make enough experimental measurements of angle pairs (δ, θ) for this purpose. Tabulate the results. (3.b) Plot the measured points on graph paper.

TASK 4. Derive the function $\delta = \delta(\theta)$ theoretically and compare it with experiment. (3 points)

(4.a) Derive a formula for $\delta = \delta(\theta)$. *Hint:* use these relations, which follow from the figure:

$$\delta = \theta + \theta' - \alpha \quad \text{and} \quad \alpha = \theta_1 + \theta_1' \qquad (3)$$

(2 points)

(4.b) Report the calculated δ values for the θ values measured in (3.a). On the graph paper used in (3.b), add the calculated δ values at the corresponding θ values. Use the refractive index *n* and angle α obtained in TASK 2. (1 point)

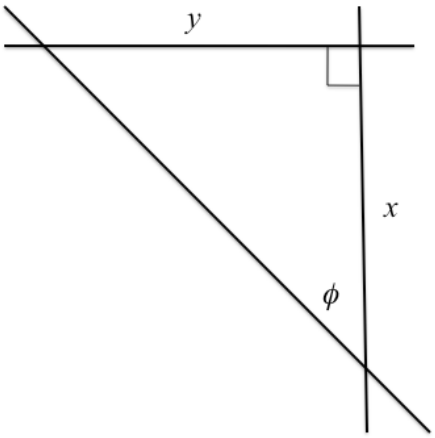
[Answer Sheet] *Experimental Problem*

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TASK 1. Use extra sheets if necessary. (3 POINTS TOTAL)

1. Draw an experimental arrangement for determining angles δ and θ.

The angle is found from $\phi = \arctan\frac{y}{x}$



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TASK 2. Use extra sheets if necessary. (8 POINTS TOTAL)

(2.a) Minimum angle (IN DEGREES)

δ_{min}
35.6
36.3
36.1
35.6
36.0

(2.b) Measurement of angle α. . You may make ONE careful measurement and assign the uncertainty from the distance measurements, or make several measurements and take their average.

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(2.c) Uncertainty analysis

For angle $\delta_{\min}$, calculate the average:

$$\overline{\delta}_{\min} = \frac{1}{5}\sum_{i=1}^{5}\delta_i = 35.9$$

Find the uncertainty from the root mean square deviation:

$$\Delta\delta_{\min} = \sqrt{\frac{1}{5}\sum_{i=1}^{5}(\delta_i - \overline{\delta}_{\min})^2} = 0.3$$

For angle α measured from distances, we obtain

α = 59.8 +- 0.5 degrees

Find individual angle uncertainties as follows. The measured lengths *x* and *y* give the angle by

ϕ = arctan (*y*/*x*)

Typical lengths average 100 mm or more, and their measurement uncertainty is

$\Delta x = \Delta y = 1.0$ mm

Thus, the uncertainty of the ratio *y*/*x* is

$\Delta(y/x) = (\Delta y/x) + y\,\Delta x/x^2 = 0.02$

This gives an uncertainty in angle ϕ (, assuming a typical angle of 45º:

$\Delta\phi = \Delta(y/x)\cos^2\phi = 0.01$ radians $= 0.01 \times 180/\pi = 0.5^\circ$

Thus, take the uncertainty of all angles measured by this method as 0.5 degrees. Therefore, angles may be given to the first decimal place, e.g. ϕ = 38.7 +- 0.5

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(2.d) Refractive index and its uncertainty.

Use the minimum-angle formula

$$n_1 = n_0 \frac{\operatorname{sen}\left(\frac{\delta_{\min} + \alpha}{2}\right)}{\operatorname{sen}\frac{\alpha}{2}} = 1.487$$

using the mean values of $\delta_{\min}$ and α, with $n_0 = 1.0$.

For the uncertainty, use the derivative formula:

$$\Delta n_1 = \frac{\cos\left(\frac{\overline{\delta}_{\min} + \overline{\alpha}}{2}\right)}{\operatorname{sen}\frac{\overline{\alpha}}{2}} \Delta\left(\frac{\delta_{\min} + \alpha}{2}\right) + \frac{\operatorname{sen}\left(\frac{\overline{\delta}_{\min} + \overline{\alpha}}{2}\right)\cos\frac{\overline{\alpha}}{2}}{\operatorname{sen}^2\frac{\overline{\alpha}}{2}} \Delta\left(\frac{\alpha}{2}\right)$$

where

$$\Delta\left(\frac{\delta_{\min} + \alpha}{2}\right) = \frac{1}{2}\sqrt{(\Delta\delta)^2 + (\Delta\alpha)^2} \approx 0.005 \text{ radians}$$

This gives

$$n_1 \approx 1.487 \pm 0.007 \approx 1.49 \pm 0.01$$

$$\alpha \approx 59.8 \pm 0.5 \quad \text{degrees}$$

SCORE:

[Answer Sheet] *Experimental Problem*

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TASK 3. Use extra sheets if necessary. (6 POINTS TOTAL)

(3.a) Measured angle pairs (δ, θ). (IN DEGREES)

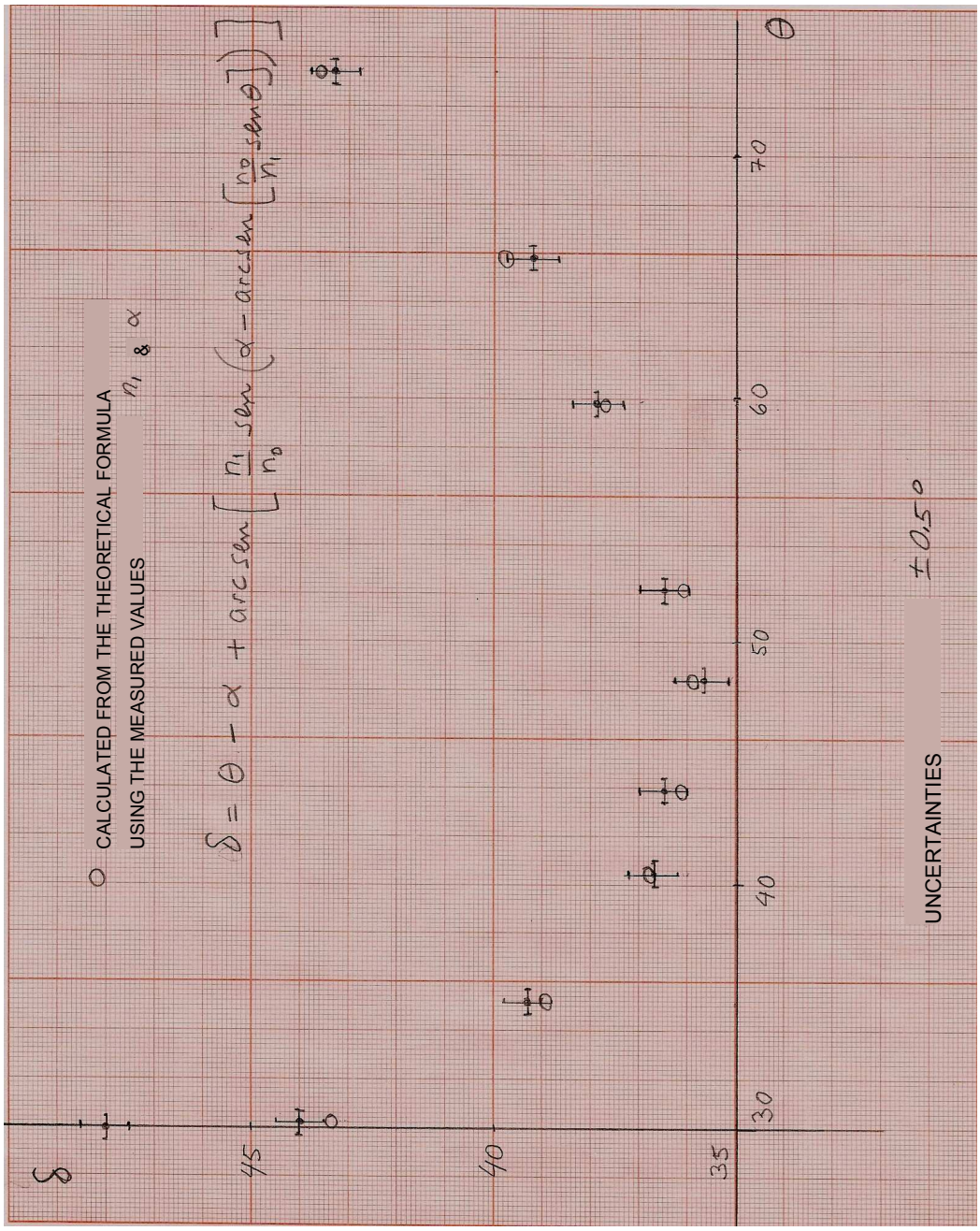
δ	θ
48.0	30.0
44.0	30.2
39.3	35.1
36.7	40.4
36.5	43.9
35.7	48.4
36.5	52.1
37.9	59.8
39.2	65.8
43.3	73.4

(3.b) All uncertainties are +- 0.5 degrees. See (2.b).

[Answer Sheet] *Experimental Problem*

Student code:	Page number:	Total pages:

(3.c) Graph of measured and calculated points (see 4.b)



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[Answer Sheet] *Experimental Problem*

Student code:	Page number:	Total pages:

TASK 4. Use extra sheets if necessary. (3 POINTS TOTAL)

(4.b) Derive the theoretical formula

Using $\delta = \theta + \theta' - (\theta_1 + \theta_1')$ and $\alpha = \theta_1 + \theta_1'$

we see that only θ' is needed. Apply Snell's law at each face:

$n_1 \text{sen}\theta_1 = n_0 \text{sen}\theta$ and $n_1 \text{sen}\theta_1' = n_0 \text{sen}\theta'$

Find θ'; substitute in δ

$$\delta = \theta + \text{arcsen}\left\{\frac{n_1}{n_0}\text{sen}\left[\alpha - \text{arcsen}\left(\frac{n_0}{n_1}\sin\theta\right)\right]\right\} - \alpha$$

(4.b) Calculate theoretical δ using θ from table (3.a).

δ_{measured}	θ	$\delta_{\text{calculated}}$
48.0	30.0	43.7
44.0	30.2	43.4
39.3	35.1	38.9
36.7	40.4	36.8
36.5	43.9	36.1
35.7	48.4	35.9
36.5	52.1	36.1
37.9	59.8	37.7
39.2	65.8	39.8
43.3	73.4	43.6

Points are in graph (3.c).

(4.c) Except for the first point, at a very small incidence angle, theory and experiment agree very well within error.

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