XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

E-1

Code

EXPERIMENTAL TEST:

ANALYSIS OF A TUBE–SPRING OSCILLATING SYSTEM

A POINT IS USED AS THE DECIMAL SEPARATOR IN THIS TEXT.

EXPRESS YOUR RESULTS IN SI UNITS.

SHEETS MARKED "E" SET OUT THE EXPERIMENTAL TASKS.

USE SHEETS MARKED "W" FOR YOUR MEASUREMENT TABLES, CALCULATIONS AND GRAPHS.

- **SHEET W-2 SHOWS A DIAGRAM OF ALL INSTRUMENTS USED IN THE EXPERIMENT.**
- **SHEET W-2a SHOWS PHOTOGRAPHS OF ALL INSTRUMENTS USED.**
- **RECORD THE GENERAL MEASUREMENTS SPECIFIED ON SHEET W-3.**
- **ENTER ALL PRINCIPAL CALCULATED RESULTS ON SHEET W-4, AS INSTRUCTED.**

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

E-2

(in Spanish)

INTRODUCTION:

Oscillations of solid bodies (pendulums, springs and pendulum–spring systems) allow us to study properties such as mass, moments of inertia, natural frequencies and resonant modes.

Springs are usually modelled as having neither mass nor moment of inertia. Here, however, the spring has non-negligible mass and therefore contributes inertia to its own oscillation and a moment of inertia to the tube–spring system. We also use a compact conical spring that obeys Hooke's law only when a certain mass is suspended from it.

First, this experiment measures the equivalent mass of a spring oscillating vertically. It then measures moments of inertia of a tube oscillating as a pendulum about an axis. Finally, it measures the moment of inertia and damping constant of the tube coupled to the spring.

IF A TASK SHEET HAS INSUFFICIENT SPACE FOR ALL INTERMEDIATE CALCULATIONS, USE THE SHEETS MARKED "W- ", ADDING THE APPROPRIATE TASK NUMBER AFTER "W- " AS SPECIFIED IN EACH TASK.

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

E-3

TASK 1- (2.0 points)

The first aim is to find the spring constant k of a conical spring extended by an attached 200 g mass (m_{ac} = 200 g), which you should treat as an intrinsic part of the spring. (This attached mass stretches the spring into the range where Hooke's law holds.)

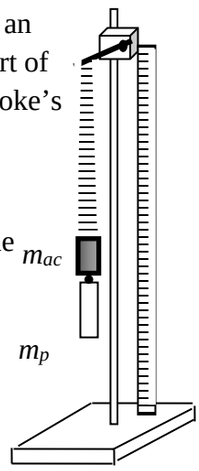


Fig. 2

Set up the apparatus in Fig. 2, with the smaller-radius end of the spring at the top. Determine k statically, without oscillations, using the available weights of mass m_p.

Use the local gravitational acceleration g = 9.78 m/s^2.

Use sheet W-5a. Label extra sheets W-5b, W5c, etc.

Enter k on sheet W-4.

Do not determine the uncertainty in k in this task.

TASK 2- (5.5 points)

The conical spring and its attached mass oscillate like a particle suspended from an ideal massless spring of spring constant k. The particle's mass is called the system's *equivalent mass* m_{eq}.

The aim is to determine this *equivalent mass* using vertical oscillations.

From your measurements, find the *equivalent mass* m_{eq} of the extended spring and determine k again. Recall that the period T of the mass–spring system is $T = 2\pi\sqrt{m/k}$, where $m = m_{eq} + m_p$.

Use sheet W-6a. **Label any extra sheets W-6b, W-6c, etc.**

Enter k and m_{eq} with their uncertainties on sheet W-4.

TASK 3- (3.0 points)

The aim is to measure the tube's principal moment of inertia I_{yy} about the *y* axis through its centre of mass (Fig. 3), **using oscillations**. The tube can oscillate as a pendulum about different axes. The pendulum period *T* and moment of inertia *I* about the suspension axis obey:

$$2\pi/T = \sqrt{mgh/I}\ ,$$

where *h* is the distance from the suspension point to the centre of mass.

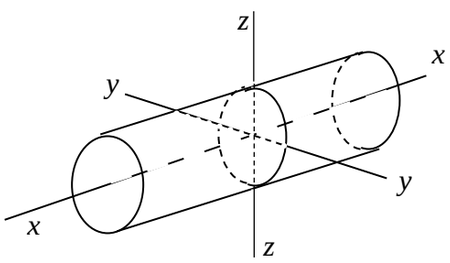


Fig. 3

Figure 3 shows three principal axes. Only the moment I_{yy} about axis *yy* is to be measured. Rods and

stands are provided for the measurements needed to calculate the moment of inertia from the tube's pendulum motion about axis *y'y'*, **parallel** to principal axis *yy* (Fig. 4).

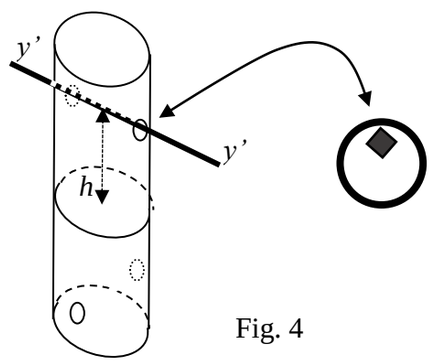


Fig. 4

After measuring $I_{y'y'}$, calculate the principal moment I_{yy} . Figure 4 shows how to position axis *y'y'*.

Use sheet W-7a for calculations. Add sheets numbered as instructed if necessary.

On W-3, enter the tube's mean outer radius R_e, mean inner radius R_i and length *L*, with estimated uncertainties.

Near one end, the tube has two holes *y'y'* through which the horizontal square-section rod can be inserted as the pendulum's suspension axis. Set up the rod carefully: keep it horizontal, secure the suspension points and minimize the contact surfaces with the axes. The tube mass m_t is written on an attached card; record it with its uncertainty on W-3, then remove the card.

Compare the experimental I_{yy} with the theoretical value:

$$I_{yy\ teor} = (m_t/4)[R_i^2 + R_e^2 + L^2/3].$$

On W-4, enter experimental $I_{y'y'}$ with its uncertainty, I_{yy} without uncertainty, $I_{yy\ teor}$ without uncertainty, and the percentage error between I_{yy} and $I_{yy\ teor}$.

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

E-5

(in Spanish)

TASK 4- (5.0 points)

Couple the tube to the vertically extended spring (spring + attached mass), as in Fig. 5, keeping the tube horizontal. See Fig. 2 on W-2 for the spring–tube coupling.

The rotational moment of inertia of the system in Fig. 5 now includes contributions from both the tube and the spring. The spring may be modelled as a particle attached to the tube; its contribution is called the *spring's equivalent moment of inertia*, I_{eq}.

The aim is to calculate this equivalent moment I_{eq} when the spring oscillates coupled to the tube about horizontal axis *y'y'* near one end. Determine the tube–spring system's moment $I_{t\text{-}r}$ experimentally, then calculate the *spring's equivalent moment* I_{eq}.

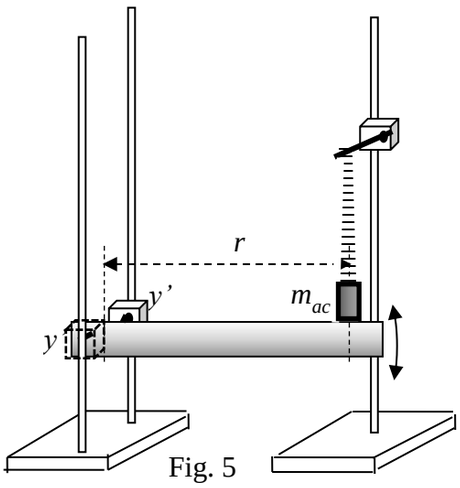


Fig. 5

Measure the oscillation period several times, and calculate the system's moment $I_{t\text{-}r}$ about axis *y'y'*.

To calculate $I_{t\text{-}r}$, use the following relation for the oscillation period:

$$2\pi/T = \sqrt{kr^2/I_{t-r}}\ ,$$

where *r* is approximately the distance from the oscillation axis to the spring attachment point.

Use the additive property of moments of inertia to obtain I_{eq} from $I_{t\text{-}r}$: $I_{t\text{-}r} = I_{yy'} + I_{eq}$.

Also calculate $I' = m_{eq}\, r^2$. I' is the theoretical spring moment if it behaves as a particle at the attachment point.

Within the experimental errors, which you must calculate, can you conclude $I' = I_{eq}$ for the extended spring, or are these independent parameters in the theoretical models of mass–spring and horizontal pendular-bar oscillations? Write your answers on W-9a.

Use W-9a, and any correctly numbered extra sheets, for measurements and calculations.

Enter I_{eq} and I' with their uncertainties, and your conclusion on whether they may be regarded as equal, on W-4.

NOTE: The holes carrying axis *y'y'* have radii much smaller than the tube length. Thus, to first approximation, $I_{y'y'}$ is practically unchanged whether the axis supports the tube at either edge of the holes, as in Figs. 4 and 5. The difference is of order 1%.

TASK 5- (3.5 points)

The fifth and final aim is to calculate the exponential damping factor of the tube–spring oscillations using the setup in Fig. 6. (Use adhesive tape to attach indicator I to the tube's end.)

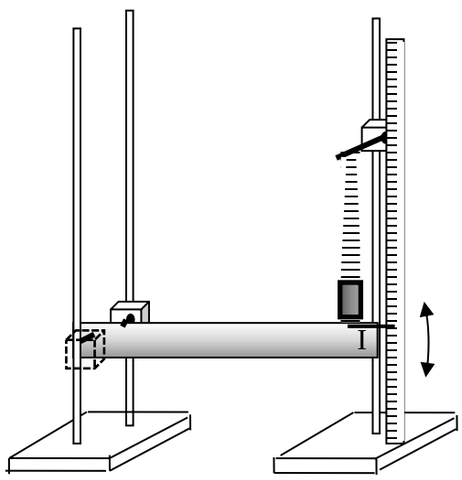


Fig. 6

With this setup, measure the free end's oscillation amplitude from 8.0 cm down to 4.0 cm. Record the time for the amplitude to fall from 8 cm to 7 cm, from 8 to 6, from 8 to 5 and from 8 to 4. Repeat the timing several times for the same intervals.

Tabulate amplitudes A and times t, and determine λ graphically, assuming the amplitude approximately follows $A = A_o e^{-\lambda t}$.

No uncertainty for λ.

Include the data table and experimental graph on W-10a.

Enter λ on W-4. _____

List of values on W-3 (1.0 point)

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

W-1

NOTE

FIRST COMPLETE ALL MEASUREMENTS, GRAPHS AND CALCULATIONS OF THE REQUIRED QUANTITIES. ONLY AT THE END CALCULATE THE REQUESTED ERRORS (OR UNCERTAINTIES).

ERROR ANALYSIS IS IMPORTANT AND FORMS PART OF THE ASSESSMENT, BUT PRIORITY WILL BE GIVEN TO EXPERIMENTAL SKILL IN OBTAINING CORRECT DATA, CORRECT GRAPH CONSTRUCTION AND MATHEMATICAL USE OF THE DATA TO CALCULATE PHYSICAL QUANTITIES.

EXTRA CALCULATION SHEETS ARE PROVIDED AT THE END. IF, FOR EXAMPLE, YOU USE EXTRA SHEETS FOR TASK 4, NUMBER THEM IN THE TOP-RIGHT CORNER W-4b, W-4c, ETC., AND PLACE THEM AFTER W-4a.

XXIII Olimpiada IBEROAMERICANA de FÍSICA

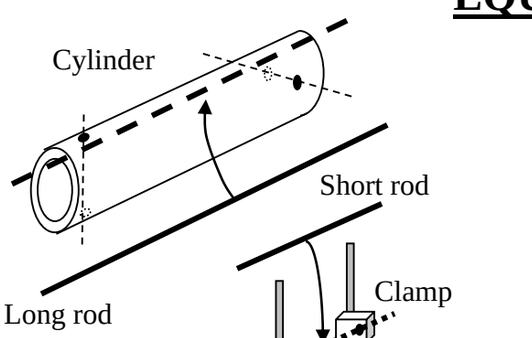
Puerto Rico 2018

Experimental Test

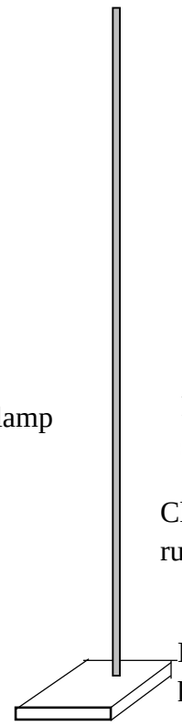
(in Spanish)

W-2

EQUIPMENT:



Two short axis-support stands

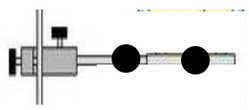


Long stand for hanging the spring

Stopwatch

Spirit level

200 g mass

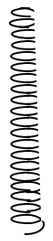


Ruler/spring support **S**



Clip securing the ruler to support **S**

Vernier

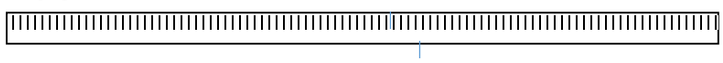


Conical spring hung on support **S**

Piece of tape

Mass hanger and masses

Wire indicator



Ruler

Fig. 1

Metal hook



Use the metal hook to couple the tube and spring

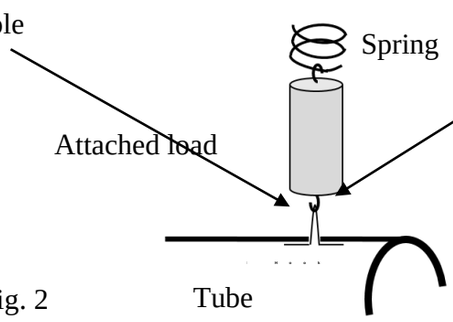


Fig. 2

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

W-2a



Cylinder



Conical spring



Two short stands to fix axes



Long stand for hanging the spring



Long rod



Short rod



Clamps



200 g mass



Ruler/spring support **S**



Spirit level



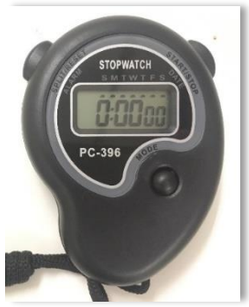
Adhesive tape



Mass hanger and masses



Vernier calliper



Stopwatch



Metal hook



Clip securing the ruler to support **S**



Wire indicator

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

W-3

RESULTS TO REPORT

(FINAL CHECKLIST)

TASK 1-

k = ____9.2 N/m _____

TASK 2-

k = ___9.2 N/m _____

δk = _____± 0.3 N/m _____

m_{eq} = ____0.23 kg _____

δm_{eq} = ___± 0.02 kg _____

TASK 3

$I_{x'x'}$ = ____0.001166 kg m^2_____

I_{xx} = _____0.00058 kg m^2_____

$I_{xx,teor}$ = ___0.0057 kg m^2_____

Dif(%)$_{exp\text{-}teor}$ between I_{xx} and $I_{xx\ teor}$ = _____1.7%_____

$I_{y'y'}$ = _____0.0197 kg m^2_____

$\delta_{y'y'}$ = _____0.0003 kg m^2 _____

I_{yy} = _____0.0072 kg m^2_____

$I_{yy,teor}$ = ____0.0066 kg m^2 _____

Dif(%)$_{exp\text{-}teor}$ between I_{yy} and $I_{yy\ teor}$ = _____9%_____

TASK 4-

$I_{t\text{-}r}$ = _____0.0534 kg m^2_____

I_{eq} = _____0.034 kgm^2_____

δI_{eq} = _____0.004 kg m^2_____

I' = _____0.0373 kgm^2_____

$\delta I'$= _____0.0005 kg m^2_____

Do your results allow $I_{eq} = I'$ to be assumed?

EQUAL: UNCERTAINTIES OVERLAP

TASK 5-

λ = _____0.020 s^{-1}_____

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

W-4

DATA SHEET; INCLUDE EACH QUANTITY'S UNCERTAINTY.

Spring mass m_r: 0.101 ± 0.001 kg

Tube mass m_t: 0.304 ± 0.001 kg

Tube length L: 0.500 ± 0.001 m

Tube outer diameter D_e: (0.088 and 0.090): 0.089 ± 0.001 m

Tube inner diameter D_i: (0.086 and 0.087): 0.086 ± 0.001 m

Tube outer radius R_e: 0.0445 ± 0.0005 m

Tube inner radius R_i: 0.0435 ± 0.0005 m

Diameter d of the side holes: 0.007 ± 0.001 m

Distance s from the tube's edge to the nearer edge of a side hole: 0.048 ± 0.001 m

Distance l from the tube's edge to the centre of the nearest side hole: 0.050 ± 0.001 m

Distance r from axis $y'y'$ to axis $z'z'$ (Fig. 4): 0.407 ± 0.001 m

Distance h from axis $y'y'$ to the tube centre: 0.200 ± 0.001 m

(If any distance requires several measurements, estimate its random error.)

YOU NEED NOT FILL EVERY ROW AND COLUMN OF THE MEASUREMENT TABLES FOR EACH TASK: USE ONLY THOSE YOU CONSIDER NECESSARY.

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

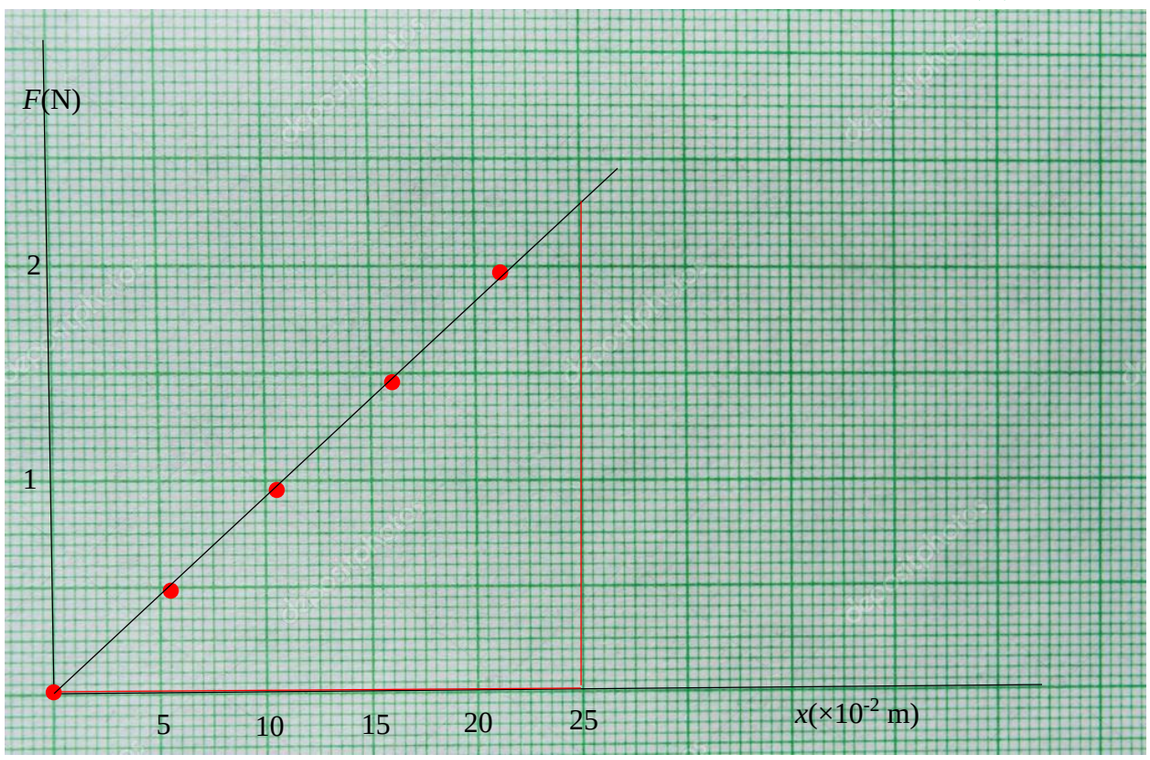
W-5a

TASK 1-

Measurement of *k* (static method):

m(g)	*x*(cm)	*Δx*(cm)	*mg*(N)			
0	41.6	0	0			
50	46.7	5.1	0.484			
100	52.1	10.5	0.979			
150	57.5	15.9	1.457			
200	62.9	21.2	1.957			

GRAPH: (1)



$k = \Delta F/\Delta x = 2.3 / (25\times10^{-2}) =$ 9.2 N/m

……..*k* : (0.5)

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

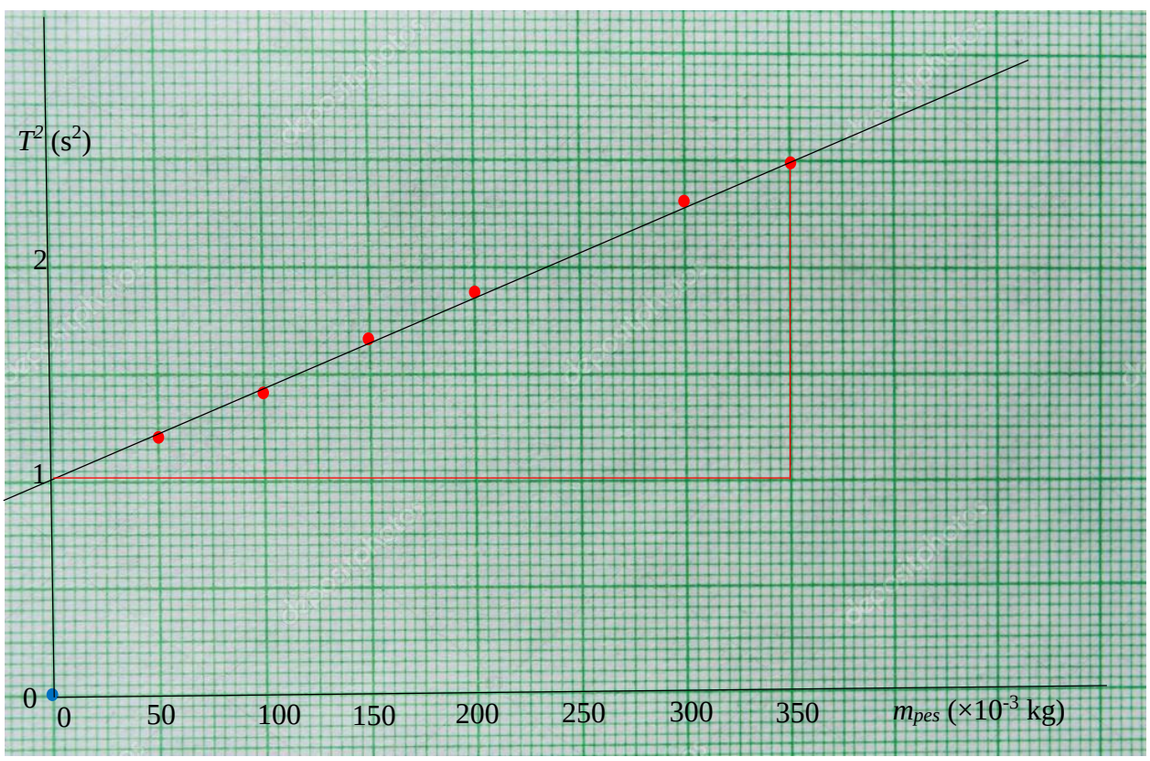
(in Spanish)

W-6a

TASK 2-

Measurement of m_{eq} and k (dynamic method): (time t for 10 oscillations)

m(g)	t_1(s)	t_2(s)	t_3(s)	t_{avg}(s)	T(s)	T^2(s^2)
50	10.97	11.00	10.94	10.97	1.097	1.203
100	11.97	12.03	11.97	11.99	1.999	1.438
150	12.93	12.94	12.96	12.94	1.294	1.674
200	13.72	13.69	13.62	13.68	1.368	1.871
300	15.28	15.21	15.28	15.26	1.526	2.327
350	15.81	15.81	15.84	15.82	1.582	2.503



$2\pi/T = [k/m]^{1/2} = [k/(m_{pes} + m_{eq})]^{1/2}$

$T^2 = (4\pi^2/k)\, m_{pes} + (4\pi^2/k)\, m_{eq}$

Slope = (2.5 – 1.0) / (0.350 – 0) = 4.286 = $4\pi^2/k$; k = 9.21 N/m

Intercept = 1.00 = $(4\pi^2/k)\, m_{eq}$; m_{eq} = 0.233 kg

By error propagation (from the graph scales):

$\delta k = k\,[\delta m_{pes}/\Delta m_{pes} + \delta T^2/\Delta T^2)] = 9.2(2/350 + 0.05/2.5) \approx$ 0.3 N/m

$\delta m_{eq} = m_{eq}\,[\delta k/k + \delta T^2/\ interc)] = 0.233(0.3/9.2 + + 0.05/1.0) \approx$ 0.02 kg

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

W-7a

TASK 3-

Measurement of I_{yy}: (timings t for 10 oscillations)

t(s)	T(s)					
11.31	1.131					
11.35	1.135					
11.34	1.134					
1132	1.132					
11.36	1.136					
			T_{avg}(s)	δT_{avg} (s)		
			1.134	±0.003		

$I_{y'y'} = mghT^2/(4\pi^2) = 0.304\times9.786\times0.203\times1.134^2 / (4\pi^2) =$ 0.0197 kg m^2 (h = 0.200 + 0.003 m)

$I_{yy} = I_{y'y'} - mh^2 = 0.0197 - 0.304\times0.203^2 =$ 0.0072 kg m^2 ≈ 0.007 kg m^2

$I_{yy\ teor} = 1/4\ m(R_e^2 + R_i^2 + L^2/2) = 0.25\times0.304(0.044^2 + 0.043^2 + 0.500^2/3)$

$I_{yy\ teor} =$ 0.0066 kg m^2 ≈ 0.007 kg m^2

$\delta I_{y'y'} = I_{y'y'}(\delta m/m + \delta g/g + \delta h/h + 2\delta T/T) =$

$= 0.0194(0.001/0.304 + 0 + 0.001/0.203 + 2\times0.003/1.134) =$ 0.0003 kg m^2

Dif(%) = (6/66)×100 = 9%

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

W-9a

TASK 4-

Measurement of I_{eq}: (timings t for 10 oscillations)

t(s)	T(s)				δT_{avg} (s)		
12.10	1.210						
11.88	1.188						
12.00	1.200						
11.91	1.191						
11.97	1.197						
				T_{avg}(s)	δT_{avg} (s)		
				1.197	±0.013		

$I_{t\text{-}r} = kr^2T^2/(4\pi^2) = 9.2\times0.400^2\times0.200\times1.197^2 / (4\pi^2) =$ 0.0534 kg m^2

But: $I_{t\text{-}r} = I_{y'y'} + I_{eq}$

$I_{eq} = I_{t\text{-}r} - I_{y'y'} = 0.0534 - 0.0197 = 0.0337$ kgm^2 = 0.034 kg m^2 $(I_{y'y'} = 0.0197 - 0.0004)$

$\delta I_{t\text{-}r} = I_{t\text{-}r}(\delta k/k + 2\delta r/r + 2\delta T/T) =$

$\delta I_{t\text{-}r} = 0.0534(0.3/9.2 + 2\times0.003/0.400 + 2\times0.013/1.197) \approx 0.004$ kg m^2

$\delta I_{eq} = \delta I_{t\text{-}r} + \delta I_{y'y'} = 0.004 + 0.0003 \approx$ 0.004 kg m^2

$I'_{eq} = m_{eq}r^2 = 0.233\times0.400^2 = 0.0373$ kg m^2 = 0.0373 kg m^2

$\delta I'_{eq} = I'_{eq}(\delta m_{eq}/m_{eq} + 2\delta r/r) = 0.037(0.002/0.233 + 2\times0.001/0.400) =$ 0.0005 kg m^2

$I'_{eq} = 0.0373 \pm 0.0005$ kg m^2

$0.030 < I_{eq} < 0.038$ and $0.0368 < I'_{eq} < 0.0378$

The two values agree within their uncertainty intervals.

XXIII Olimpiada IBEROAMERICANA de FÍSICA

Puerto Rico 2018

Experimental Test

(in Spanish)

W-10a

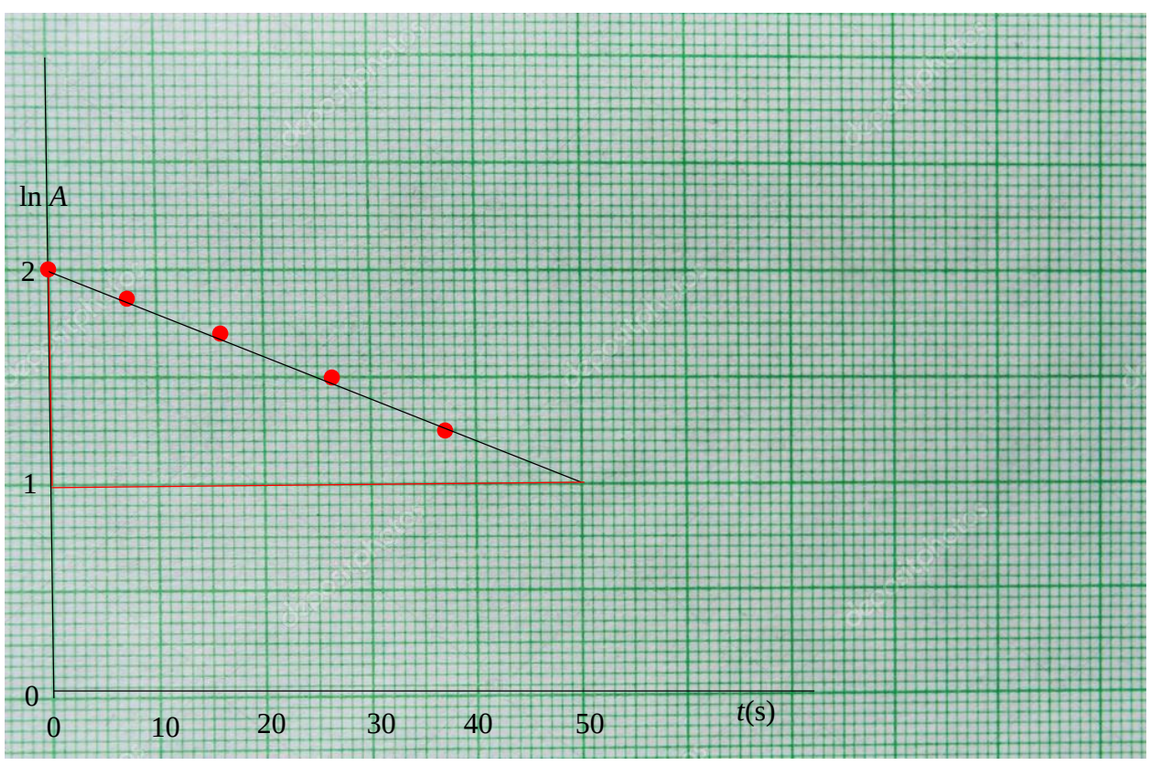
TASK 5-

Measurement of λ.

Equilibrium position on the ruler: x_o = 30.5 cm

x(cm)	t_1(s)	$t2_1$(s)	t_3(s)	t_{avg}(s)	A(cm)	ln A
38	0	0	0	0	7.5	2.01
37	8.28	6.19	7.35	7.2	6.5	1.87
36	17.47	14.35	15.78	15.8	5.5	1.70
35	26.51	26.66	25.36	26.2	4.5	1.50
34	37.50	36.75	36.41	36.9	3.5	1.25

λ = 0.020 s^{-1}



λ =(2 -1) / (50 – 0) = 1/50 = 0.020 s^{-1} $A = 7.5(\text{cm})\, e^{-.020t}$