Surface-tension reduction and ecosystems

Introduction

Surface tension is a physicochemical phenomenon linked to molecular interactions in a fluid. It results from increased energy at the interface between two fluids. The system tends towards equilibrium at the lowest-energy configuration, changing its geometry to reduce the interfacial area.

Surface-active agents, also called surfactants, reduce surface tension at a surface or between two liquids.

Detergent pollution also affects solid waste: many wastes become less able to float, disrupting ecosystems and causing living organisms to disappear. Human waste affecting sea turtles and penguins is a controversial example, though clearly not the only cause.

This experiment studies the effect of soap concentration in water using a particular surface-tension phenomenon: **capillarity**. A fluid rises in a capillary because of surface tension, which depends on fluid cohesion and enables it to rise or fall along a narrow tube.

A liquid rises in a capillary when intermolecular cohesion is weaker than adhesion to the tube material: the liquid wets the surface. It rises until surface tension balances the weight of liquid in the tube (Figure 1). This occurs with water and partly regulates its ascent inside plants without spending energy to overcome gravity. Reduced surface tension alters this property and therefore affects organisms.

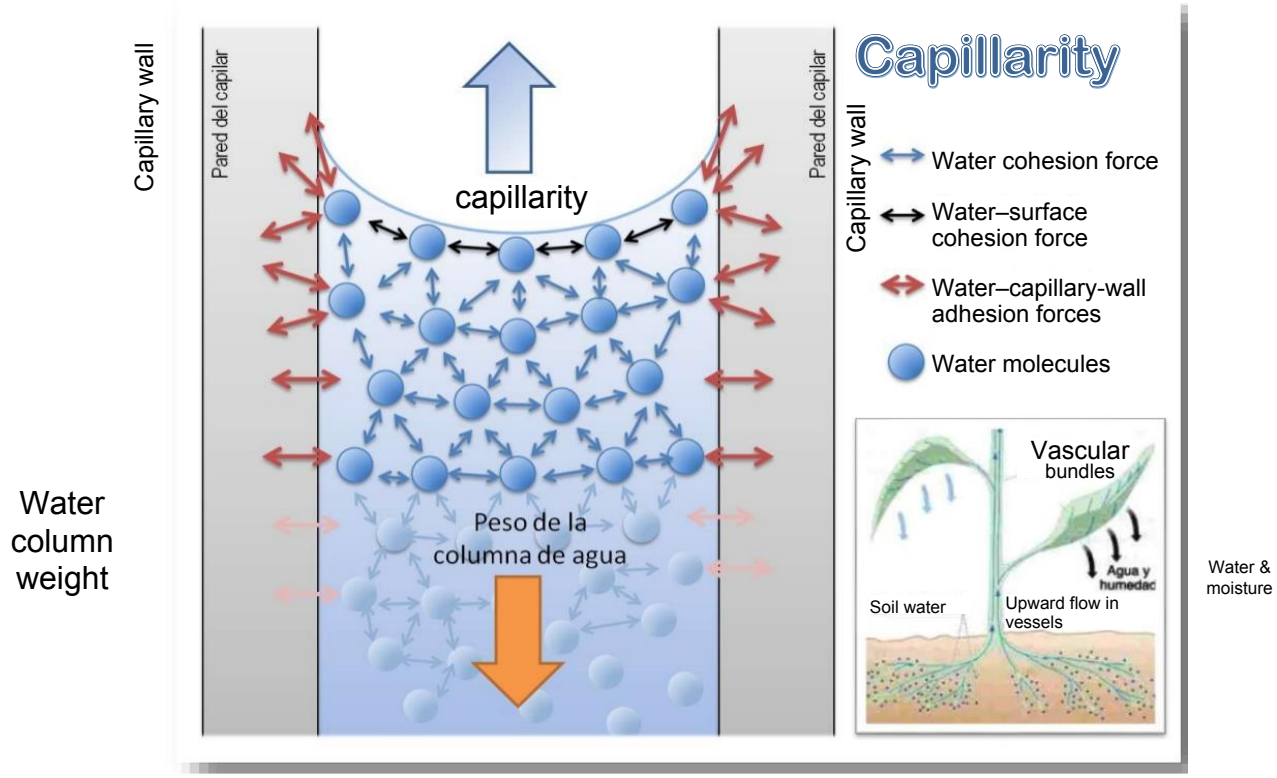


Figure 1: Surface-tension forces at capillary walls and in a plant.

Capillarity measures surface tension by immersing a glass tube in a solution and then withdrawing it. The retained liquid-column height gives the surface tension coefficient σ.

The equilibrium condition for the liquid in the tube is

$$\frac{2\sigma}{r} = \rho g h \tag{1}$$

Here ρ is liquid density and r the capillary radius. This condition gives the surface tension coefficient σ through

Let ρ_0 and σ_0 be the density and surface tension coefficient of pure water, and h_0 its column height. The surface tension coefficient of the soap solution can then be calculated from

$$\sigma = \sigma_0 \frac{\rho h}{\rho_0 h_0} \tag{2}$$

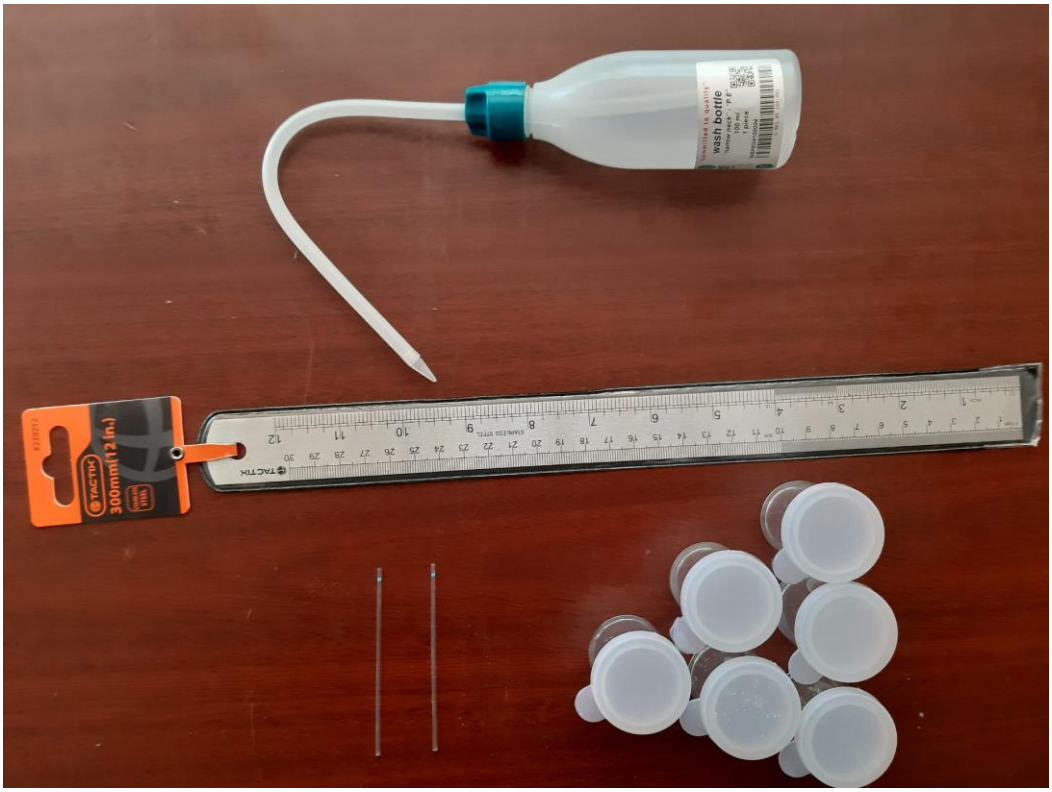
Because soap concentrations are very low, approximate the solution density as constant. Thus:

$$\sigma = \sigma_0 \frac{h}{h_0} \tag{3}$$

Use $\sigma_0 = 72.75 \times 10^{-3}$ N/m and $h_0 = (2.60 \pm 0.05)$ cm at (25 ± 2) °C and atmospheric pressure.

Equipment

- 2 capillary tubes
- 7 bottles of soap–water solutions with different concentrations
- Graduated ruler
- 1 wash bottle or small air pump
- Graph paper



Part A. Concentration dependence

Study how liquid surface tension depends on surfactant concentration, specifically the surface tension of soap–water solutions at different soap concentrations. Seven solutions have been prepared (Table A). Use a capillary tube to calculate their surface tensions. Measurement procedure:

1. Draw a marker line on the tube to show the depth immersed in the solution. The blue band on the tube can serve as a guide.

2. Insert and shake the tube in the solution to wet its interior. Withdraw it and blow it empty with the wash bottle. Repeat at least 3 times to condition it to each solution.

3. Cover the upper end with a finger and immerse the tube in the solution. This prevents bubbles forming inside.
4. Uncover the tube end and wait until the solution stops rising.
5. Withdraw the tube and wait until the liquid stops descending owing to changes in pressure and viscosity.
6. Cover the same upper end and carefully bring the tube near the ruler without contact. Measure the liquid-column height.

Before measuring, observe the following recommendations:

- Random errors in capillary-column height make repeated measurements essential; repeat as often as you judge necessary.
- After each measurement, use the wash bottle (pump) to blow the tube clean so no drops remain inside.
- Before measuring a concentration, immerse and blow the tube to condition it to that solution and prevent mixing with drops of other concentrations.

- Do not dry the tube with your shirt: it could break.
- **Note:** the second tube is a spare only. Each tube is different.

A. 1	Tabulate concentrations and capillary heights with their errors. Label this Table A1.	**2.5pt**

A. 2	Calculate surface tension coefficient σ for each concentration c and plot the corresponding graph.	**2 pt**

Experiment shows surface tension depends on soap concentration; express this relation as

(4) $$\sigma = Ac^b$$

Here c is soap concentration, and A and b are parameters to be determined.

A. 3	Linearize the graph from A.2.	**2.0 pt**

A. 4	Calculate parameter b with its uncertainty.	**1.0 pt**

Part B. Practical application

Statistics allows interpolation and extrapolation to predict outcomes from known variables. Use the Part A data to characterize soap–water solution X of unknown concentration.

B. 1	From its measured capillary height, determine the surface tension of solution X with uncertainty.	**0.5 pt**

B. 2	Use Part A results to determine the concentration of solution X with uncertainty.	**2.0 pt**

Table A1

Table A

Solution	C (% m/v)
A	0.0021
B	0.0142
C	0.0383
D	0.0800
F	0.2507
H	0.8009

Surface-tension reduction and ecosystems
Solution

Part A. Concentration dependence

A. 1 Tabulate concentrations and capillary heights with their errors. Label this Table A1. **2.5pt**

Table A1 (1.5pts)

C [gr/ml]	h1 [cm]	h2 [cm]	h3 [cm]	h4 [cm]	h5 [cm]	h mean [cm]	Error
0.0021	2.25	2.35	2.35	2.35	2.25	2.31	0.02
0.0142	2.00	1.85	1.80	1.80	1.75	1.84	0.04
0.0383	1.80	1.70	1.60	1.60	1.55	1.65	0.04
0.0800	1.75	1.55	1.50	1.45	1.45	1.54	0.06
0.2507	1.35	1.35	1.20	1.10	1.15	1.23	0.05
0.8009	1.15	1.10	1.10	1.05	1.00	1.08	0.03
1.6004	1.00	1.00	0.95	1.00	1.00	0.99	0.01

Average at least five measurements to obtain the capillary height.

For each concentration, the height error can be obtained as the standard error of the mean.

$$\Delta h = \sqrt{\frac{\sigma}{n}}$$

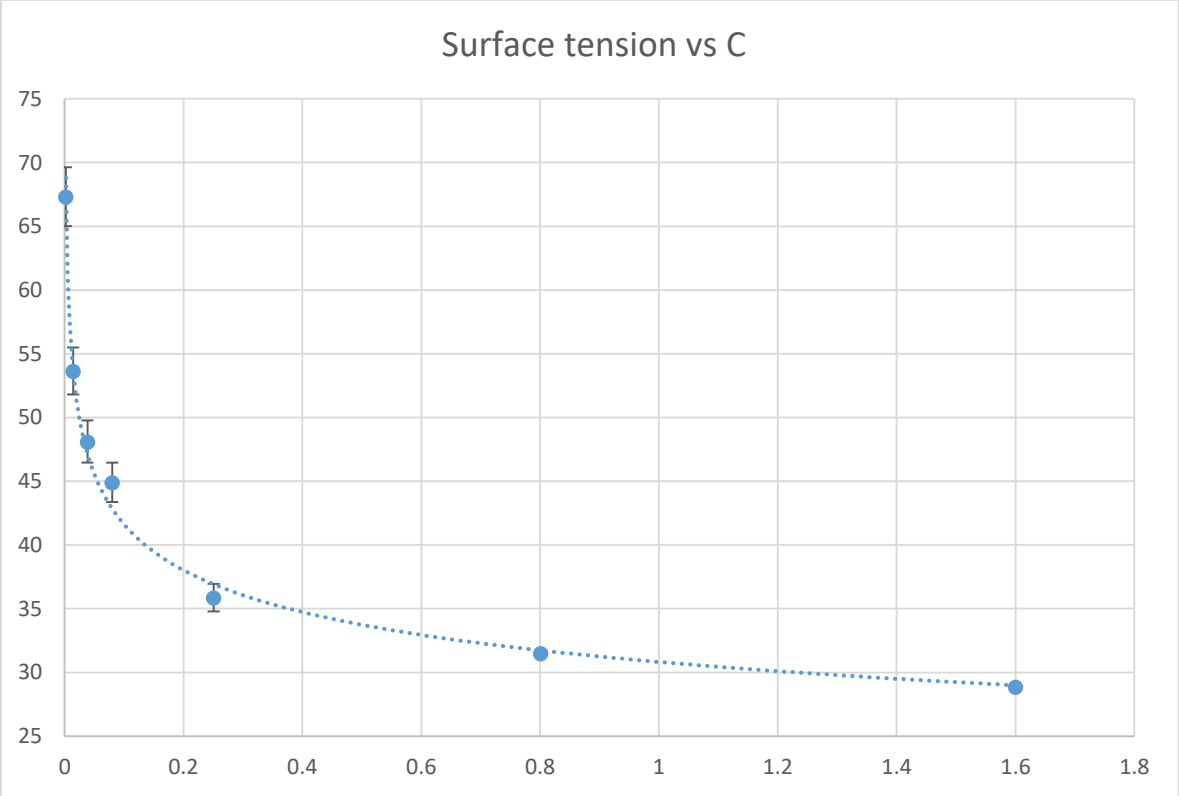
Here σ is the sample standard deviation of the heights. (1.0pt)

A. 2 Calculate surface tension coefficient σ for each concentration c and plot the corresponding graph. **2 pt**

Table A2 (1.0pt)

C [gr/ml]	h mean [cm]	Error	sigma (dyn /cm)
0.0021	2.31	0.02	67.30
0.0142	1.84	0.04	53.60
0.0383	1.65	0.04	48.07
0.0800	1.54	0.06	44.86
0.2507	1.23	0.05	35.83
0.8009	1.08	0.03	31.46
1.6004	0.99	0.01	28.84

(1.0pt)

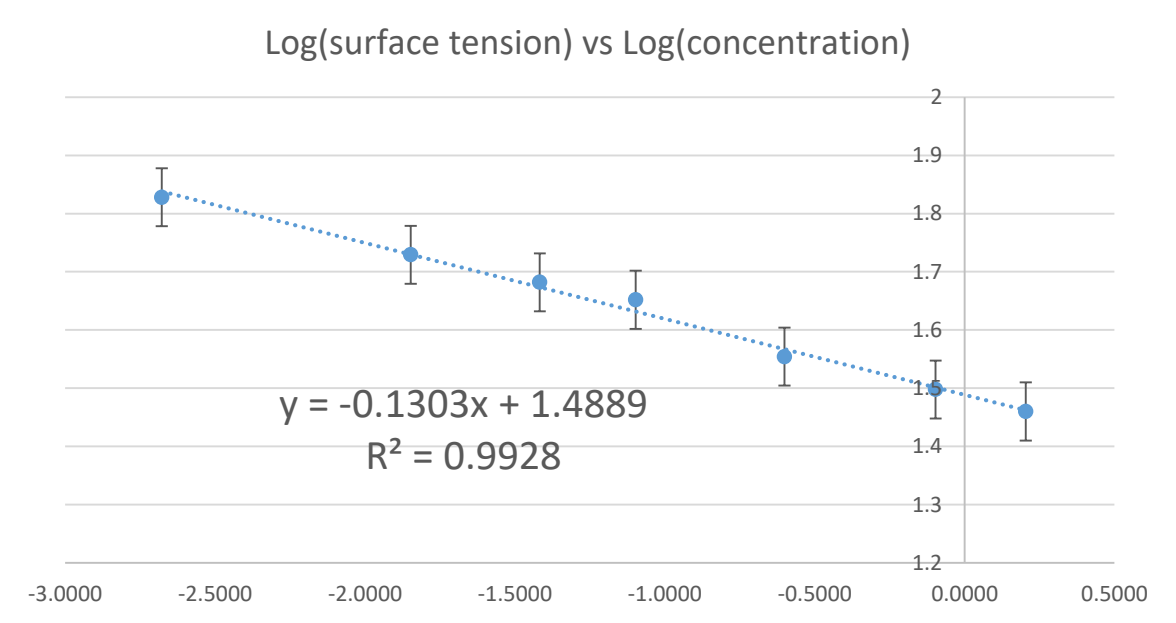


A. 3 Linearize the A.2 graph. **2.0 pt**

(1.0pt)

C [gr/ml]	Log(C)	sigma (dinas/cm)	Log(sigma)
0.0021	-2.6778	67.306	1.83
0.0142	-1.8477	53.60	1.73
0.0383	-1.4168	48.07	1.68
0.0800	-1.0969	44.86	1.65
0.2507	-0.6008	35.83	1.55
0.8009	-0.0964	31.46	1.50
1.6004	0.2042	28.84	1.46

(1.0pt)



A. 4	Calculate parameter *b* with its uncertainty.	**1.0 pt**

The slope may be determined graphically by measuring it directly on the A3 graph. (0.5pts)

Estimate the slope uncertainty by drawing maximum- and minimum-slope lines. (0.5pts)

$$b = (-13.03 \pm 0.49) \times 10^{-2}$$

Alternatively use least squares to find the parameter and the slope's standard error. (0.5pts)

$$b = \frac{cov(x,y)}{\sigma_x^2}$$

Here σ_x^2 is the variance of *x*; cov(x,y) is the covariance of *x* and *y*. (0.5pts)

$$\Delta b = \frac{\sigma_y}{\sqrt{\sum_i (x_i - \bar{x})^2}}$$

Here σ_y is the sample standard deviation of *y*.

Part B. Practical application

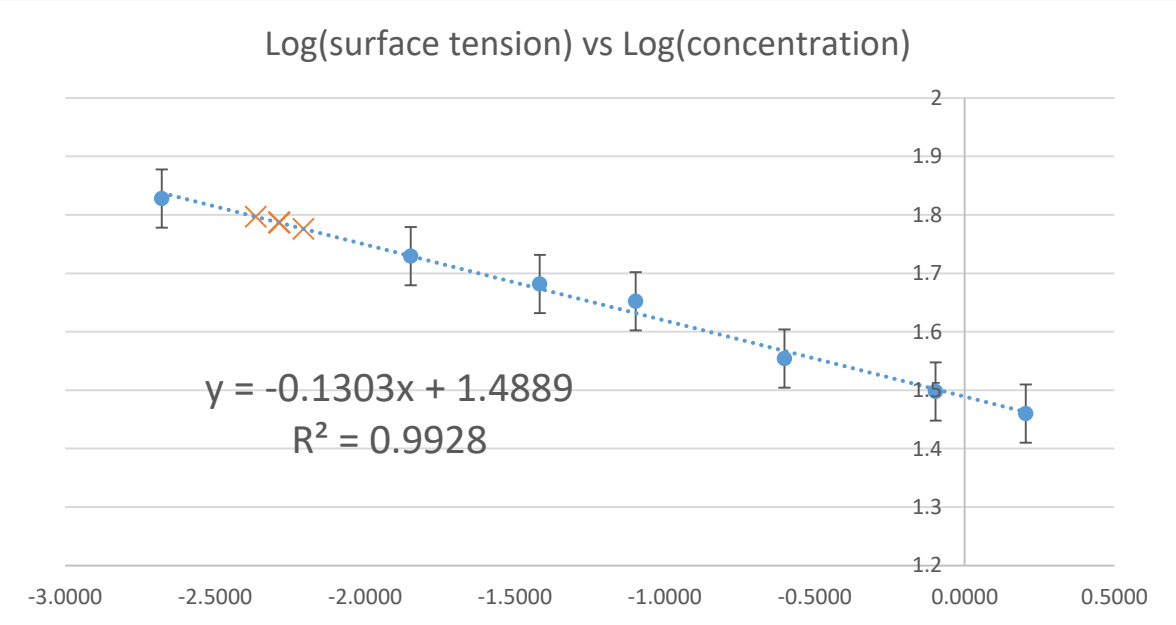
B. 1	From its measured capillary height, determine the surface tension of solution X with uncertainty.	**0.5 pt**

h1 [cm]	h2 [cm]	h3 [cm]	h4 [cm]	h5 [cm]	h avg. [cm]	Error
2.15	2.1	2.1	2.05	2.1	2.1	0.01

Repeat capillary-height measurements and take the mean. (0.25pts)

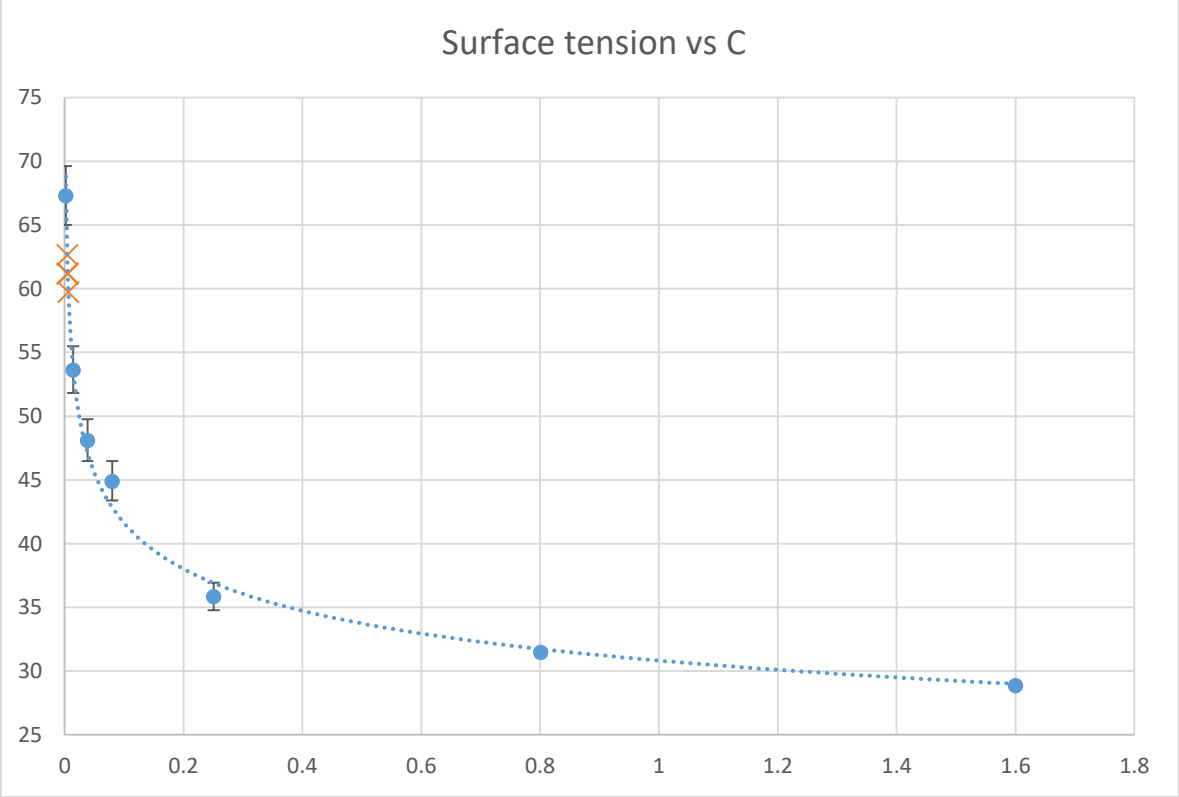
Again, use the standard error of the mean as the uncertainty. (0.25pts)

B. 2	Use Part A results to determine the concentration of solution X with uncertainty.	2.0 pt



(1.0pts)

Solution	C (% m/v)
A	0.0021
B	0.0142
C	0.0383
D	0.0800
F	0.2507
H	0.8009
J	1.6004



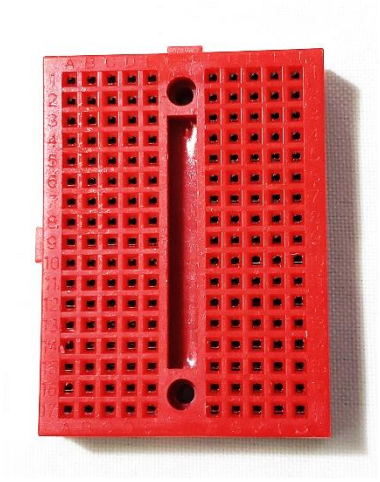
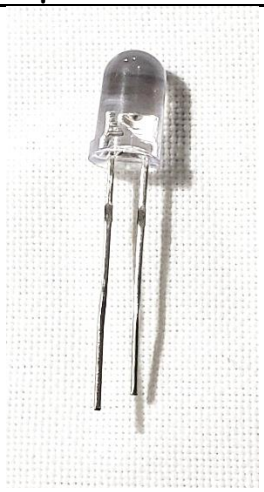
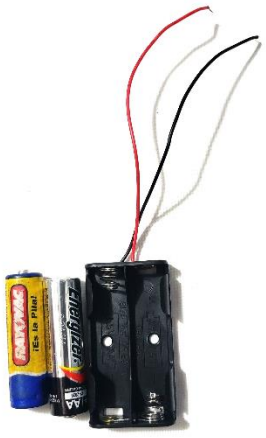
$$C_x = (5.1 \pm 0.9) \times 10^{-3}\%(m/v)$$

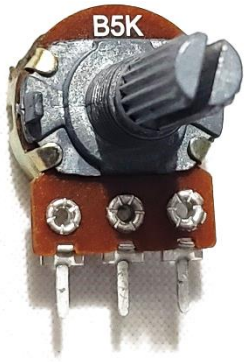
(1.0pts)

Estimate the error as the interval containing all concentrations.

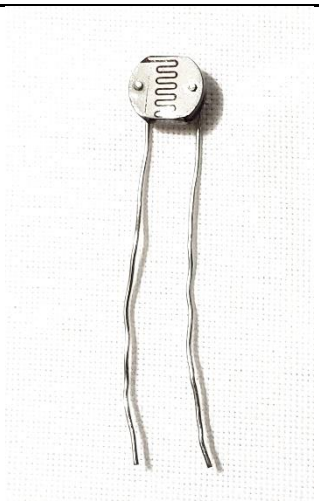
LED and photoresistor characterization (10 points)

Equipment:

 Breadboard or protoboard.	 Connecting leads for the breadboard.	 Crocodile-clip leads.
 Red LED.	 Battery holder with batteries	 Two digital multimeters



A variable resistor or rheostat (potentiometer).



A photoresistor

Unlike other commercial breadboards, the one used here has only two columns of rows with five holes each. The five holes in each row are connected internally; each row is independent of the others.

Light-emitting diodes (LEDs) are promising high-intensity light sources with efficiencies at least ten times those of traditional sources. Applications range from signalling, sensing and indicators to backlighting computer and mobile-device displays and general illumination.

Commercial LEDs of different colours are encapsulated in epoxy resin, with two terminals for the voltage V that enables light emission. The component has polarity, as shown in Figure 1(a).

With an efficient light source, a device for estimating emitted-light intensity is useful. An early such device is the photoresistor, whose resistance is inversely proportional to incident-light intensity. It is also called a photoconductor, photoelectric

cell or light-dependent resistor (LDR). Its structure is shown in Figure 1(b).

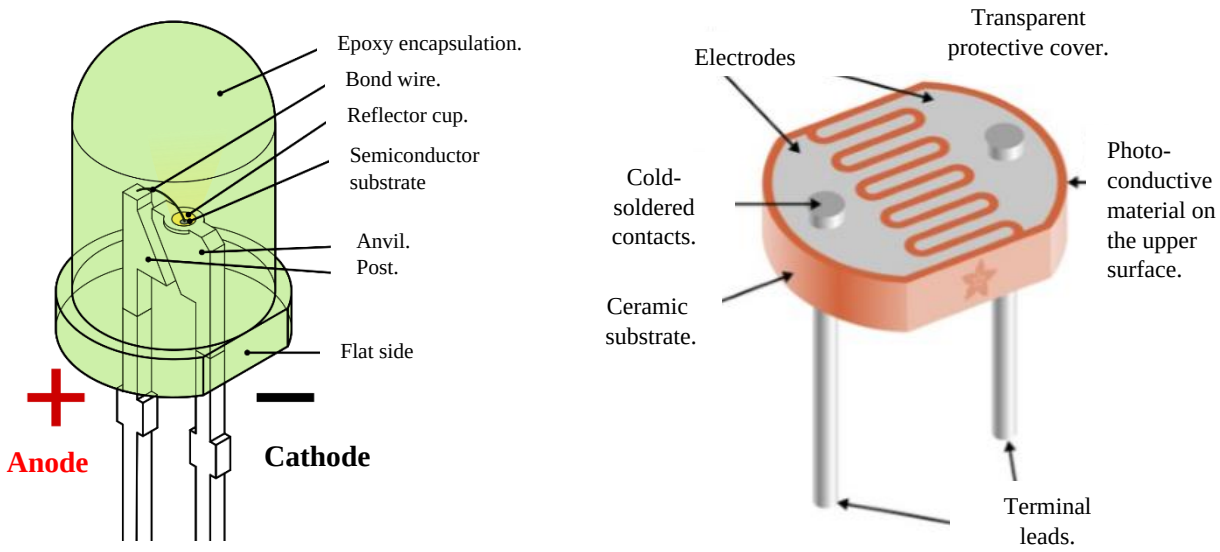


Figure 2: (a) LED diagram showing its components and terminal-lead polarity [1]. (b) Photoresistor diagram showing its components and nonpolarized terminals [2].

Part A: LED characteristic curve.

Study the nonlinear dependence of LED current on the voltage applied across its terminals. Measure both terminal voltage and current. First assemble the circuit in Figure 3 using the supplied components, observing the indicated polarity.

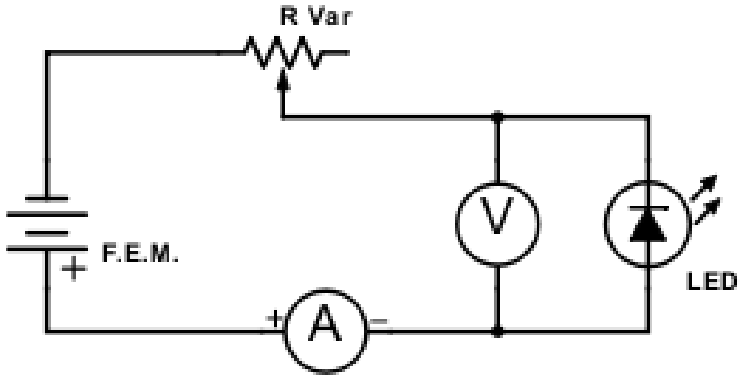


Figure 3: Circuit for Part A. The EMF source, ammeter A, LED and variable resistor R Var are in series, with the indicated polarities. Voltmeter V is parallel to the LED.

Before assembling the apparatus and measuring, read these recommendations:

- Ensure the work area is dry and free of soap residue.
- Assemble all components and ensure there is no short circuit before connecting the batteries (EMF).
- Because of its sensitivity, the ammeter MUST NOT be connected directly to measure battery current. This may blow its internal protective fuse and make current measurement impossible.
- Check that the digital multimeter is set to the correct mode and range for the required quantities.
- Disconnect the batteries after measuring to avoid discharging them prematurely.
- The potentiometer acts as a variable resistor when its centre terminal and either side terminal are connected. Leave the other side terminal unconnected.
- Multimeters work as voltmeters or ammeters depending on the central selector setting. Choose the range carefully.

A. 1	Tabulate LED current and terminal voltage. Take enough measurements for statistical analysis.	**2.0 pt**

Experiment gives the following current–voltage relation:

$$I = I_1 e^{\beta V} + I_0 \qquad (1)$$

. Thus voltage depends logarithmically on current as

$$V = V_1 \log(I) + V_0 \qquad (2)$$

A. 2	Find parameters V_0 and V_1 in equation 2, with their uncertainties.	**3.0 pt**

Part B: Characterizing a photoresistor using the LED.

Study the photoresistor and its relation to LED light intensity. Assume the diode efficiently converts the electrical power supplied into light intensity:

$$\text{Intensity} \propto \text{Power} \quad .$$

This relation is valid within our measurement range.

With a variable-intensity source, controlled by varying LED power with the potentiometer, find the relation between light intensity and photoresistor resistance. Assemble Figure 4, observing the indicated polarity.

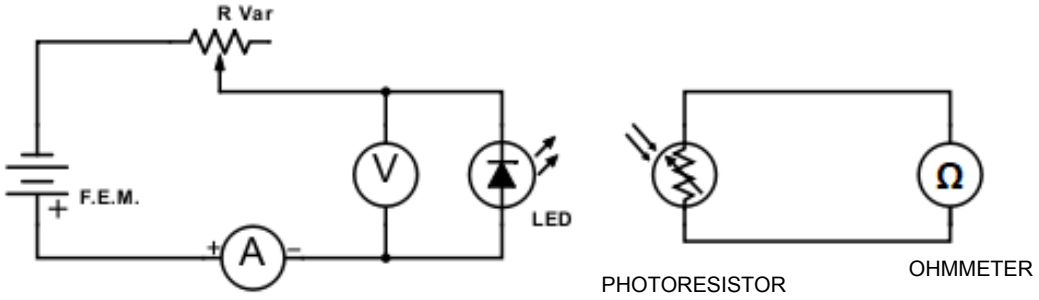


Figure 4: Circuit for Part B. The EMF source, ammeter A, LED and variable resistor R Var are in series with the indicated polarities. A multimeter in ohmmeter mode measures the photoresistor's effective resistance.

Before assembling the apparatus and measuring, read these recommendations:

- Ensure the work area is dry and free of soap residue.
- Place the photoresistor close to the LED so it receives mainly LED light rather than ambient light.
- Assemble all components and ensure there are no short circuits before connecting the batteries (EMF).
- Because of its sensitivity, the ammeter MUST NOT be connected directly to measure battery current. This may blow its internal protective fuse and make current measurement impossible.
- Check that the digital multimeter is set to the correct mode and range for the required quantities.
- Disconnect the batteries after measuring to avoid discharging them prematurely.

- The potentiometer acts as a variable resistor when its centre terminal and either side terminal are connected. Leave the other side terminal unconnected.

- Multimeters work as ohmmeters or ammeters depending on the central selector setting. Choose the range carefully.

B. 1	Tabulate LED current against measured photoresistor resistance. Take enough measurements for statistical analysis.	**1.0 pt**

To determine LED intensity we need its emitted power. Use the A.2 graph and equation (2) to calculate terminal voltage, then calculate power from

$$P = I\,V \;\rightarrow f(I) \qquad (4)$$

. Thus power can be expressed solely as a function of current.

B. 2	Tabulate measured LED current, calculated terminal voltage, LED power at that current and measured photoresistor resistance.	**2.0 pt**

B. 3	Use Table B.2 to find the relation between LED power and photoresistor resistance.	**2.0 pt**

[1] Safa Kasap. Electrical Engineering Department. University of Saskatchewan. Canada.

[2] GL55 Series Photoresistor Datasheet. CdS Manual.

LED and photoresistor characterization (10 points)

A. 1	Tabulate LED current and terminal voltage. Take enough measurements for statistical analysis.	**2.0 pt**

Red LED	
I (mA)	**V (Volts)**
0.75	1.802
0.8	1.807
0.89	1.813
0.93	1.815
1.08	1.825
1.19	1.832
1.26	1.836
1.39	1.845
1.51	1.852
1.6	1.857
1.66	1.859
1.72	1.863
1.81	1.866
1.85	1.87
2.02	1.875
2.15	1.876
2.31	1.883
2.4	1.881
2.7	1.901
2.47	1.892
2.86	1.907
2.96	1.909
3.42	1.923
3.97	1.939
3.99	1.938
5.7	1.98

5.84	1.983
6.01	1.986
6.4	1.968

A. 2	Find parameters V_0 and V_1 in equation 2, with their uncertainties.	**3.0 pt**

Change the variable from $I \rightarrow \ln I$ and construct the following table:

Red LED		
I [A]	**ln(I)**	**V [Volt]**
0.00075	-7.19544	1.802
0.0008	-7.1309	1.807
0.00089	-7.02429	1.813
0.00093	-6.98033	1.815
0.00108	-6.83079	1.825
0.00119	-6.7338	1.832
0.00126	-6.67664	1.836
0.00139	-6.57845	1.845
0.00151	-6.49565	1.852
0.0016	-6.43775	1.857
0.00166	-6.40094	1.859
0.00172	-6.36543	1.863
0.00181	-6.31443	1.866
0.00185	-6.29257	1.87
0.00202	-6.20466	1.875
0.00215	-6.14229	1.876
0.00231	-6.07051	1.883
0.0024	-6.03229	1.881
0.0027	-5.9145	1.901
0.00247	-6.00354	1.892
0.00286	-5.85693	1.907
0.00296	-5.82257	1.909

0.00342	-5.67811	1.923
0.00397	-5.52899	1.939
0.00399	-5.52396	1.938
0.0057	-5.16729	1.98
0.00584	-5.14302	1.983
0.00601	-5.11433	1.986

Statistical analysis of the preceding table using ln(I) and voltage gives:

Regression Statistics	
Multiple R	0.993233254
R Square	0.986512297
Adjusted R Square	0.985993539
Standard Error	0.006182345
Observations	28

	Coefficients	*Standard Error*	*Lower 95%*	*Upper 95%*
Intercept	2.42441	0.01256	2.39859	2.45022
X Variable 1	0.08792	0.00202	0.08378	0.09207

The required values are therefore:

$$V_0 = 2.42441 \pm 0.01256$$

$$V_1 = 0.08792 \pm 0.00202$$

The 95% confidence intervals for V_0 and V_1 are thus:

$$2.39859 \leq V_0 \leq 2.45022$$

$$0.08378 \leq V_1 \leq 0.09207$$

B. 1	Tabulate LED current against measured photoresistor resistance. Take enough measurements for statistical analysis.	**1.0 pt**

I [mA] (Meas.)	Resistance (Meas.)
0.00020	103000.0
0.00023	101900.0
0.00025	101000.0
0.00030	99000.0
0.00032	98100.0
0.00035	96700.0
0.00041	94800.0
0.00044	93900.0
0.00053	91300.0
0.00064	89600.0
0.00071	87500.0
0.00076	84700.0
0.00083	84200.0
0.00102	80000.0

B. 2	Tabulate measured LED current, calculated terminal voltage, LED power at that current and measured photoresistor resistance.	**2.0 pt**

I [mA] (Meas.)	Voltage [V] (Calculated)	Power [mW] (Calculated)	Resistance (Meas.)
0.00020	1.76741	0.00036	103000.0

0.00023	1.77776	0.00040	101900.0
0.00025	1.78561	0.00044	101000.0
0.00030	1.80219	0.00054	99000.0
0.00032	1.80821	0.00058	98100.0
0.00035	1.81616	0.00064	96700.0
0.00041	1.82911	0.00074	94800.0
0.00044	1.83726	0.00082	93900.0
0.00053	1.85246	0.00098	91300.0
0.00064	1.86942	0.00119	89600.0
0.00071	1.87865	0.00133	87500.0
0.00076	1.88506	0.00143	84700.0
0.00083	1.89265	0.00157	84200.0
0.00102	1.91104	0.00195	80000.0

B. 3	Use Table B.2 to find the relation between LED power and photoresistor resistance.	**2.0 pt**

Red LED				
I [mA]	**Voltage [V]**	**Power [mW]**	**Resistance**	**1/R**
(Meas.)	**(Calculated)**	**(Calculated)**	**(Meas.)**	
0.00020	1.76741	0.00036	103000.0	9.709E-06
0.00023	1.77776	0.00040	101900.0	9.814E-06
0.00025	1.78561	0.00044	101000.0	9.901E-06
0.00030	1.80219	0.00054	99000.0	1.010E-05
0.00032	1.80821	0.00058	98100.0	1.019E-05
0.00035	1.81616	0.00064	96700.0	1.034E-05
0.00041	1.82911	0.00074	94800.0	1.055E-05
0.00044	1.83726	0.00082	93900.0	1.065E-05
0.00053	1.85246	0.00098	91300.0	1.095E-05
0.00064	1.86942	0.00119	89600.0	1.116E-05

0.00071	1.87865	0.00133	87500.0	1.143E-05
0.00076	1.88506	0.00143	84700.0	1.181E-05
0.00083	1.89265	0.00157	84200.0	1.188E-05
0.00102	1.91104	0.00195	80000.0	1.250E-05

From this table, statistically analyse the power and 1/R columns:

Regression Statistics	
Multiple R	0.996433
R Square	0.992879
Adjusted R Square	0.992286
Standard Error	7.6E-08
Observations	14

	Coefficients	*Standard Error*	*Lower 95%*	*Upper 95%*
Intercept	9.17E-06	4.43E-08	9.08E-06	9.27E-06
X Variable 1	0.001741	4.26E-05	0.001648	0.001834

LED power is directly proportional to $1/R$, hence inversely proportional to R. The regression coefficients are:

$$\frac{1}{R} = C_1 * I + C_0$$

The values are $C_1 = 0.00171$ and $C_0 = 9.17 \times 10^{-6}$. Their 95% confidence intervals are:

$$0.001648 \leq C_1 \leq 0.001834$$

$$9.08 \times 10^{-6} \leq C_0 \leq 9.27 \times 10^{-6}$$

With $R^2 = 0.992879$, the fit is appropriate for this data set.