Experimental Problem: Rutherford scattering

Iberoamerican Physics Olympiad

2022

1. Introduction

Studies of atomic composition began around the turn of the twentieth century. Sir Joseph John Thomson had discovered the electron, a tiny, light, negatively charged particle present in all matter. Models of atomic structure were proposed: since atoms are neutral, they must also contain positive charge equal to the negative charge of their electrons.

One leading model was the Thomson model (1900), proposed by William Thomson with support from J.J. Thomson. Electrons were embedded in a uniform positively charged mass. It was called the *plum pudding* model, by analogy with raisins scattered through a uniform pudding.

The *Rutherford scattering experiment* was devised by Ernest Rutherford to test whether the Thomson model, or another existing model, correctly described the atom. Charged particles are fired at a target, and their scattering through Coulomb interaction with the target atoms is observed.

Ernest Marsden and Hans Geiger performed the Rutherford experiments in 1908 to 1910 under supervision from Rutherford. In the final version, alpha particles from radon-222 decay formed a narrow collimated beam incident on a 2 μm gold foil. A fluorescent screen flashed when struck by an alpha particle. Source, foil and screen were enclosed in an evacuated sealed tube.

From the results, Rutherford proposed his 1911 model: electrons orbit a compact, massive positive nucleus like planets around the Sun. It was therefore called the *planetary model of the atom*.

2. Experimental setup

Simulate a beam of 10^8 alpha particles incident on a thin gold foil. The beam area is 1 mm^2. Particle paths are exactly parallel, with random distances from the beam centre (Figure 1). Particles of energy 5.5 MeV strike the 10^{-6} m foil normally.

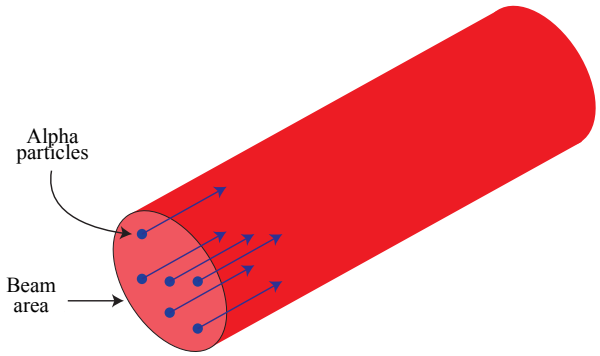


Figure 1: Alpha-particle beam. All particles have the same kinetic energy and parallel paths. the distance of each path from the beam centre is random.

The simulation accounts for the interactions of each particle with the target material to determine its trajectory on reaching the target.

A detector counts alpha particles scattered through angle Θ from the incident beam. Its circular effective area is 4 cm^2. It moves on a circle centred at the point of impact of the beam, in 15^o steps from 0^o to 135^o (Figure 2).

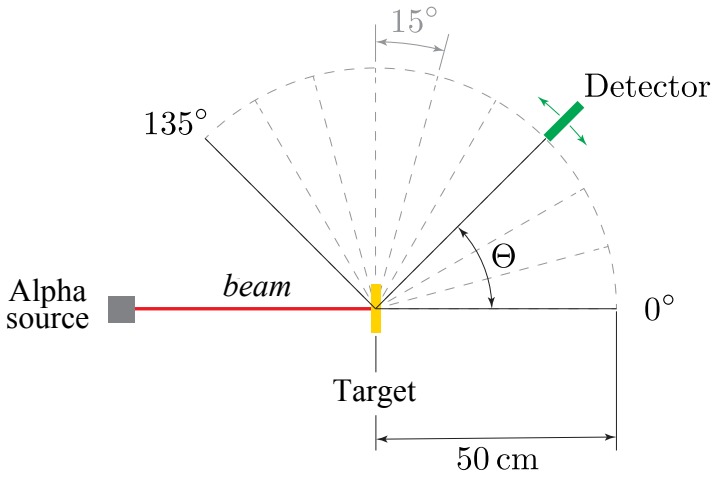


Figure 2:Rutherford scattering: Simulation setup.

Open the simulator in a web browser. no installation is needed. Find it on the Olympiad Moodle page or follow this link:

Rutherford scattering simulator.[1]

Choose the detector angle with the − and + buttons. The angle is shown graphically and numerically between them. Press ▸ to run a simulation and obtain a count. Press ▸ again for another run at the same angle.

"Delete" removes only the last count, useful if unsure whether ▸ produced a new result. "Reset" restores the initial simulator settings.

3. Experimental procedure

Use the simulator to:

1. Measure the scattered-particle count at every available detector angle.

2. Plot detector count against angle on semi-logarithmic axes. Neglect angle uncertainty.

4. Rutherford model

Rutherford concentrated all positive charge in tiny, massive nuclei. Electrons are small, so most atomic volume is empty and most particles pass without interaction. Those passing near a nucleus are scattered by Coulomb repulsion. Figure 3 shows the *hyperbolic* path of an alpha particle: the incident line is a distance b from a parallel through the nucleus, the closest approach is r, and the outgoing direction makes angle Θ with the incident beam. Accounting for nuclear charge and proportionality to the incident particle count, Rutherford derived the expected number I at angle Θ:

[1] Full address: https://oibf-2022.gofisica.org/U2FsdGVkX19tLoU3wB4bwMEQzQZH3HhQVzpN0pcjTZJfNUAX8q3Fb4TUZyHQ-qZx/main_es.html

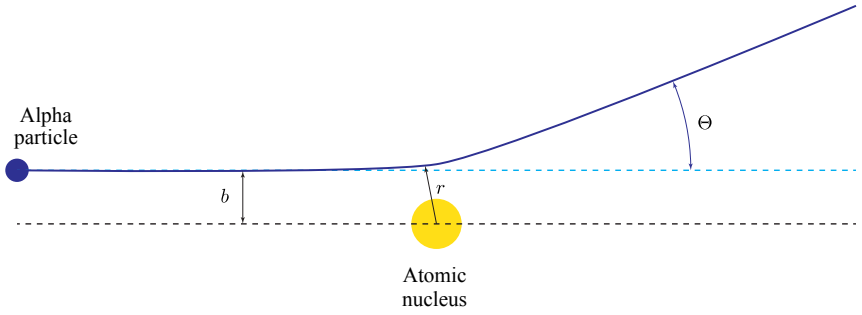


Figure 3: Rutherford model. The hyperbolic path of an alpha particle electrically repelled by a nearby atomic nucleus.

$$I = I_0 N_x \left[\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{4E}\right]^2 \frac{1}{\sin^4(\Theta/2)}\Delta\Omega. \qquad (1)$$

where:

Z_1:	Alpha-particle atomic Z (helium nuclei).	$Z_1 = 2$.
Z_2:	Gold atomic Z.	$Z_2 = 79$.
e:	Elementary charge.	$e = 1.6 \times 10^{-19}$ C.
ε_0:	Vacuum permittivity.	$\varepsilon_0 = 8.8 \times 10^{-12}$ F/m.
E:	Incident-beam particle energy.	
$\Delta\Omega$:	Detector solid angle.	
N_x:	Scattering nuclei per unit area.	
I_0:	Incident particle count.	

4.1. Fitting the Rutherford function

Equation (1) has the form:

$$I = \frac{k}{\sin^n(\Theta/2)}$$

Fit the simulator data to this function. Neglect angle uncertainties.

1. Find k.
2. Find n.

3. Can the Rutherford model explain the behaviour of scattered particles?
   Discuss briefly.

4.2 Estimating the nuclear radius

Use the simulator data to find the tightest upper bound on the nuclear radius, with these hyperbolic-trajectory expressions:

$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{2E}\sqrt{\frac{1+\cos\Theta}{1-\cos\Theta}}$$

$$r = \frac{b\cos(\Theta/2)}{1-\sin(\Theta/2)}$$

Here b is the *impact parameter*. Do not neglect angle uncertainty.

Experimental problem solution

Iberoamerican Physics Olympiad 2022

3. Experimental procedure (6 points)

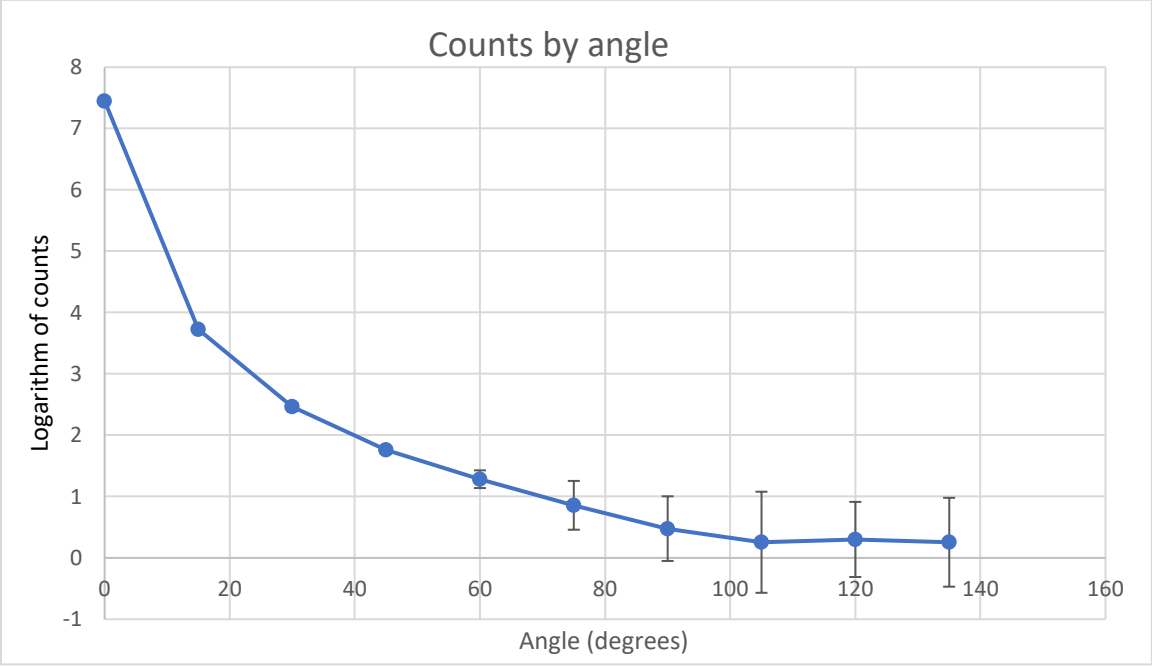
1. *Measure the scattered-particle count at every possible detector angle in the experiment.* ***(3 points)***

   Take at least 5 measurements per angle to find a mean and an uncertainty. Use either the standard deviation or the deviation of the mean, which is the standard deviation divided by the number of measurements.

2. *Construct a graph of the measurements using a semi-logarithmic scale for the particle count in the detector as a function of angle. Neglect the uncertainty in the angle.* ***(3 points)***

   Use the natural logarithm of the mean count, $Z = \ln I$,
   and propagate uncertainty using

$$\Delta Z = \frac{\Delta \mathrm{I}}{I}$$



4.1 Fitting the Rutherford function (11 points)

1. Find k. ***(4 points)***

Several fitting methods are possible. For example, use:

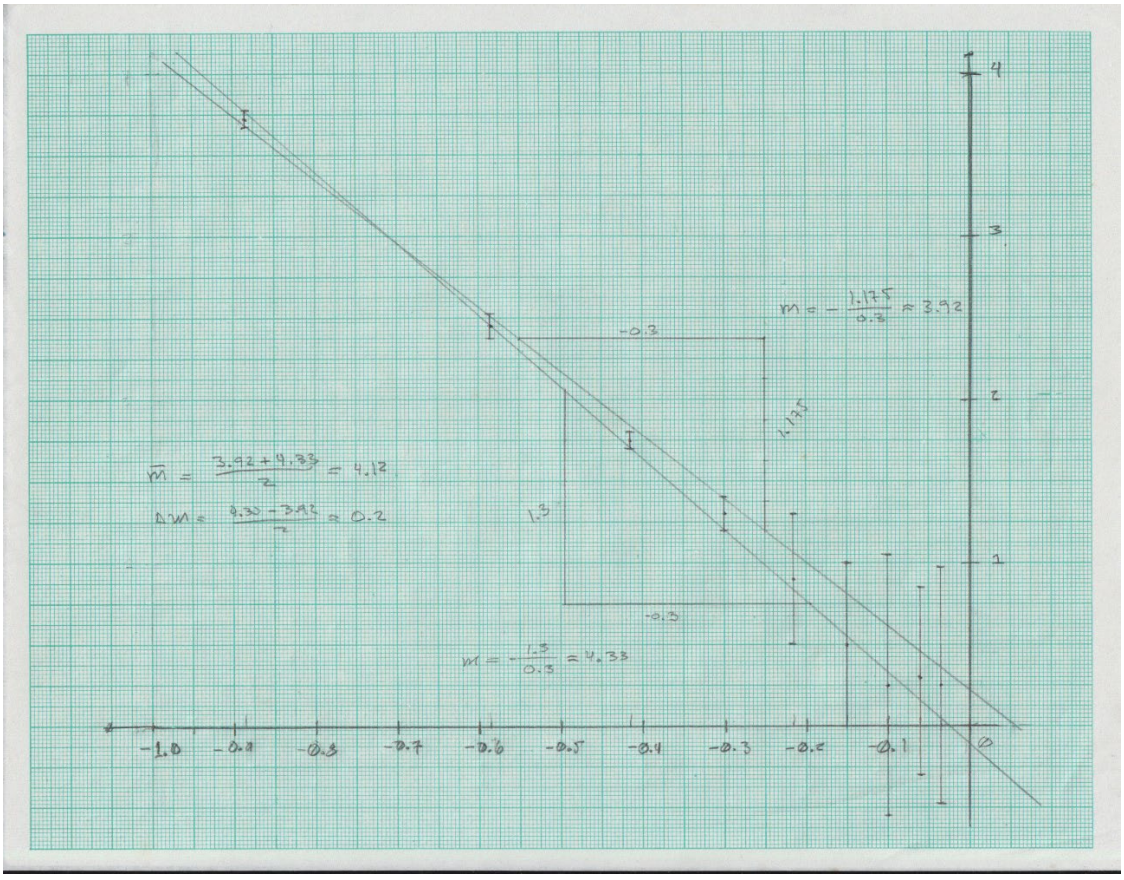
$$\ln I = \ln k - n \ln[\sin(\Theta/2)]$$

Plot on graph paper and draw two lines passing through most points within their uncertainties. This gives $k \approx 1.1$ with $\Delta k \approx 0.2$. The units of *k* are counts, or dimensionless.

2. Find n. ***(5 points)***

Similarly, $n \approx 4.1$ and $\Delta n \approx 0.2$.

This is an example graph. The x axis plots $\sin(\Theta/2)$, so the slope is negative.



3. Can the Rutherford model explain the scattered particles? Discuss briefly. ***(2 points)***

The exponent $n = 4.1 \pm 0.2$ found above agrees with the prediction of Rutherford, so the model describes the observed scattered particles.

4.2 Estimating the nuclear radius (3 points)

To find the tightest upper bound on nuclear radius from these expressions,

$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{2E}\sqrt{\frac{1+\cos\Theta}{1-\cos\Theta}}$$

$$r = \frac{b\cos(\Theta/2)}{1-\sin(\Theta/2)}$$

evaluate them using the given data and the largest angle with detected counts, here Θ=135°.

For angle uncertainty, the detector has circular area 4 cm² and lies 50 cm from the scattering centre. Its angular aperture is 2.58°, giving an angle range of 133.71° to 136.29°.

Evaluating r at these limits gives:

$b(133.71°) \approx 8.8882 \times 10^{-15}$ m, $r(133.71°) \approx 4.3405 \times 10^{-14}$ m

$b(136.29°) \approx 8.3397 \times 10^{-15}$ m, $r(136.29°) \approx 4.3196 \times 10^{-14}$ m

The mean and uncertainty of r are:

$$\bar{r} \approx 4.33 \times 10^{-14}\ \text{m}, \quad \Delta r \approx 0.01 \times 10^{-14}\ \text{m}$$