3 Simulation Lab

23. Spinny Spin Spin

It is well known that the spin of an electron is related to its magnetic moment by $\boldsymbol{m} = -g\mu_B \boldsymbol{S}$, where μ_B is the Bohr magneton, g is a dimensionless factor that depends on the electron's environment, and $\boldsymbol{S}$ is a vector of magnitude $\frac{1}{2}$. The angular momentum of the electron is given by $\hbar \boldsymbol{S}$.

Now, an ensemble of $N = 100000$ electron spins are distributed roughly evenly along a rod of length 2. The rod points in the x direction. The left end of the rod is given the coordinate $\boldsymbol{\xi} = -1$ and the right end of the rod is given the coordinate $\boldsymbol{\xi} = 1$. The magnetic field along the rod points in the z direction and is given by $B_z = B_0 + B_1\xi$. It is known that B_0 is on the order of $0.01T$ and $B_1 \ll B_0$. You can also add a bias magnetic field $\boldsymbol{B} = b\hat{z}$ which is uniform, but due to budget constraints this magnetic field must be limited to a magnitude of $0.01T$.

The spins are placed in a thermal bath and allowed to thermalise. In this initial stage, the spin will either align with the magnetic field or antiparallel to it according to the Boltzmann distribution. At time $t = 0$, a strong magnetic field pulse is applied to the system to rotate all spins 90° anticlockwise about the y axis. You can then set a sensor to detect the total magnetisation around the x and y axes at certain times (for convenience it will be given in units of μ_B), but remember these measurements are subject to noise! Also, a protection circuit is in place to prevent the initial magnetic field pulse from destroying the measurement circuit, so nothing will be recorded before $t = 100$ns. After $t = 10000$ns the measurement circuit runs out of memory and nothing will be recorded either.

Your goal is to find g, B_0 to 5% accuracy, as well as B_1 to 15% accuracy. Give the magnetic fields in units of Tesla. You may assume the spins do not interact with the environment or with each other after $t = 0$.