Light reveals the intimate structure of matter: its polarization records both the asymmetry of a medium and the geometry of an interface. — C. T. Dominguez, 2026. Free translation: Light reveals the intimate structure of matter: its polarization records both the asymmetry of a medium and the geometry of an interface.

Experimental Physics Examination and Solutions[†]

Polarization of Light in Transparent Media

rotation of the polarization direction in a sugar solution and polarization by reflection from a glass plate

Examination topics: light polarization, optical rotation and the Brewster condition.
Suggested total score: 20 points.
Suggested examination time: 5 hours.
Permitted materials: pen, pencil, eraser, ruler, graph paper and a non-programmable scientific calculator.

General Instructions

1. Read the whole examination before measuring; divide your time between the two experimental parts.
2. Record all readings, units, instrument ranges, uncertainties and relevant qualitative observations.
3. Graphs must identify the axes, units and legend, and represent the experimental data appropriately.
4. Maintain the angular references defined in each part. Do not arbitrarily redefine zero during a measurement series.
5. Before and after each series, check that the beam is aligned with the centres of the optical elements and the photodetector. Do not directly compare currents obtained with different lengths or configurations.
6. **In both parts of the examination, always assemble the kit with the laser on the right** (the beam travelling from right to left), as viewed by the experimenter working in front of the laboratory bench. This arrangement reduces the risk of reflections reaching the experimenter, but does not remove the need to control reflected beams.
7. **Strictly follow the laser safety instructions and WEAR the protective goggles (PPE) supplied by the examination staff throughout assembly and data acquisition.**
8. Handle the sugar solution and glass plate carefully. Immediately dry any liquid spilled on the laboratory bench.

LASER SAFETY: wearing the protective goggles (PPE) supplied by the examination staff is MANDATORY. Keep the goggles correctly positioned during alignment and measurements. Even with PPE, NEVER look directly into the laser beam, DO NOT point it at other people, and avoid specular reflections at eye level. Goggles reduce the risk of accidental exposure, but do not make direct viewing of the beam safe.

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Aim and Background

Polarization describes the direction in which the electric field of a light wave oscillates. When polarized light interacts with matter, its polarization direction may change during propagation, or it may be reflected and transmitted differently at an interface. This examination investigates these two effects experimentally.

Examination structure. You are not expected to know all the concepts presented here beforehand. Each stage briefly introduces an idea, relates it to what you will observe on the laboratory bench, and sets a task. Read each guided explanation before taking the corresponding measurement: the examination is designed to assess, but also to teach and practise, experimental research procedures.

How your answers will be assessed. Marking is based exclusively on the answers submitted on paper. Assembly and adjustment instructions guide your work; carrying them out earns no points by itself. For each item, present the requested evidence: labelled diagrams, readings with units and uncertainties, tables, calculations, graphs or data-based justifications. Show numerical substitutions in calculations; for repeated operations, one complete example and a table of results suffice. Statements such as "I assembled the apparatus correctly" or "the alignment was good" do not replace this evidence. If you cannot obtain a measurement, record the difficulty and analyse the available data. Safety precautions remain mandatory throughout.

Part I studies **rotation of the plane of polarization in optically active media**: the ability of certain media to rotate the polarization direction as light propagates through them. This can occur in crystals, liquids and solutions with suitable microscopic structures. An aqueous sugar solution provides an accessible, safe example of an optically active medium. Sugar molecules are chiral: they cannot be superimposed on their mirror images.[1] The observed rotation can depend on the substance and its concentration, the light path length, wavelength and temperature.

Part II studies reflection of polarized light at an air–glass interface. Components polarized perpendicular and parallel to the plane of incidence have different reflection coefficients. For parallel polarization, there is an incidence angle at which reflection ideally vanishes: the Brewster angle. Measuring this angle experimentally allows the refractive index of glass to be estimated.[2]

Initial Background: Concepts Needed for the Experiment

This section contains only the relations needed to interpret the measurements in Q1 and Q2.

[1] A chiral molecule and its mirror image form distinct structures called enantiomers. A sample containing an excess of one enantiomer may exhibit net optical rotation.

[2] The refractive index of a medium is $n = c/v$, where c is the speed of light in vacuum and v is its phase velocity in the medium.

Polarization and Malus's Law

Light is a transverse wave: its electric field $\vec{E}$ oscillates perpendicular to the propagation direction $\vec{k}$. In linearly polarized light, $\vec{E}$ oscillates in a definite direction. A polarizer mainly transmits the component of $\vec{E}$ parallel to its transmission axis.

As the analyser rotates, the current measured by the photodiode follows a shifted Malus law:

$$i(\theta_A) = a + b\cos^2(\theta_A - \phi)\,.$$

Here, θ_A is the analyser angle reading; a is the minimum signal level, b its amplitude, and ϕ the angular position of the curve. The model repeats every 180°, so all comparisons must retain the same zero, angular direction and branch of the curve.

Optical Rotation

An optically active solution rotates the polarization direction. Experimentally, this appears as a horizontal shift of the Malus curve. If ϕ_{ref} corresponds to the empty cell and ϕ_L to a solution length L, the rotation is

$$\Delta\phi(L) = \phi_L - \phi_{\text{ref}}.$$

Amplitudes need not be compared: the relevant quantity is the angular shift between the curves, with its sign and uncertainty.

This examination tests the model

$$\Delta\phi(L) = \alpha(\lambda, T)\, L^n C,$$

where C is the concentration, n the length exponent, and $\alpha(\lambda, T)$ characterizes the medium at the wavelength and temperature used. During the measurements, C, λ and T must remain constant.

Reflection and the Brewster Condition

The incident ray and surface normal define the plane of incidence. In s polarization, the electric field is perpendicular to this plane; in p polarization it is parallel.

The reflected currents i_s and i_p depend on the incidence angle θ_i. Near the Brewster angle, reflection of the p component reaches a minimum. The ratio

$$q(\theta_i) = \frac{i_s}{i_p}$$

increases in this region, but should be calculated only when i_p is positive and distinguishable from zero within its uncertainty. Determine θ_B by comparing the minimum of i_p with the region of the largest reliable ratio.

For incidence from air into glass,

$$\tan\theta_B = \frac{n_{\text{vidrio}}}{n_{\text{aire}}}, \qquad n_{\text{vidrio}} = n_{\text{aire}}\tan\theta_B.$$

Angles are measured from the normal. In this examination, use $n_{\text{aire}} = 1{,}000$ and obtain the index uncertainty by evaluating the expression at $\theta_B - u_\theta$ and $\theta_B + u_\theta$.

Instrumental Error Limit of the Multimeter

For current measurements in Q1 and Q2, use the **DT8380B** multimeter specifications in section 3.2 of the manufacturer's manual. The table below gives the **DC current** ranges used in this examination, originally in Portuguese.

Range	**Resolution D**	**Specified accuracy**
200 µA	100 nA	±(1,0 % of reading + 2D)
2000 µA	1 µA	±(1,0 % of reading + 2D)
20 mA	10 µA	±(1,0 % of reading + 2D)
200 mA	100 µA	±(1,2 % of reading + 2D)

The parameter D is the resolution of the selected range.[3] The manual's term +2 means two counts of the least significant displayed digit, i.e. $2D$, not two percent of the reading. For an indicated current i, calculate the instrumental limit from

$$\Delta i_{\text{inst}} = a_{\text{inst}}\,|i| + 2D, \qquad a_{\text{inst}} = \begin{cases} 0{,}010, & \text{in ranges } 200\,\mu\text{A},\ 2000\,\mu\text{A} \text{ and } 20\,\text{mA}; \\ 0{,}012, & \text{in range } 200\,\text{mA}. \end{cases}$$

The coefficient a_{inst} is the percentage in the table expressed as a decimal. Use the same unit for i and D; in the lowest range, $D = 100\,\text{nA} = 0{,}1\,\mu\text{A}$. Here, Δi_{inst} is the error limit specified by the manual's accuracy specification, not the standard deviation of repeated readings.[4]

Example for the DT8380B. For a reading of 4,58 mA in the 20 mA range, we have $a_{\text{inst}} = 0{,}010$, $D = 0{,}01\,\text{mA}$ and

$$\begin{aligned} \Delta i_{\text{inst}} &= 0{,}010 \times 4{,}58\,\text{mA} + 2 \times 0{,}01\,\text{mA} \\ &= 0{,}0458\,\text{mA} + 0{,}0200\,\text{mA} = 0{,}0658\,\text{mA}. \end{aligned}$$

Rounding this limit upward to the decimal place of the resolution, the reading may be reported as

$$i = (4{,}58 \pm 0{,}07)\,\text{mA},$$

considering only the **instrumental contribution**.

For N repeated readings x_k, the **sample standard deviation** s and the **standard deviation of the mean** $s_{\bar{x}}$, also used here as the statistical uncertainty of the mean u_{estat}, are given by

$$\bar{x} = \frac{1}{N}\sum_{k=1}^{N} x_k, \qquad s = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N}(x_k - \bar{x})^2}, \qquad s_{\bar{x}} = u_{\text{estat}} = \frac{s}{\sqrt{N}}.$$

Application to repeated measurements. In Q1–A.1 and Q2–A.1, calculate Δi_{inst} for every reading using a_{inst} and D for its range. Report the mean, sample standard deviation and standard deviation of the mean separately for each set of three measurements; in this case, $N = 3$ and $u_{\text{estat}} = s/\sqrt{3}$.
Do not divide Δi_{inst} by $\sqrt{3}$: repeated readings do not automatically reduce the instrumental error limit. Even when the display reads zero, the expression gives $\Delta i_{\text{inst}} = 2D$.

[3] Resolution is the smallest distinguishable change in a reading; for a digital multimeter it is the last displayed digit's step in the selected range. Resolution alone does not represent the entire specified error limit.

[4] An error limit bounds the error interval permitted by the specification. It is not, by itself, a statistical uncertainty of the mean or a standard uncertainty equivalent to one standard deviation.

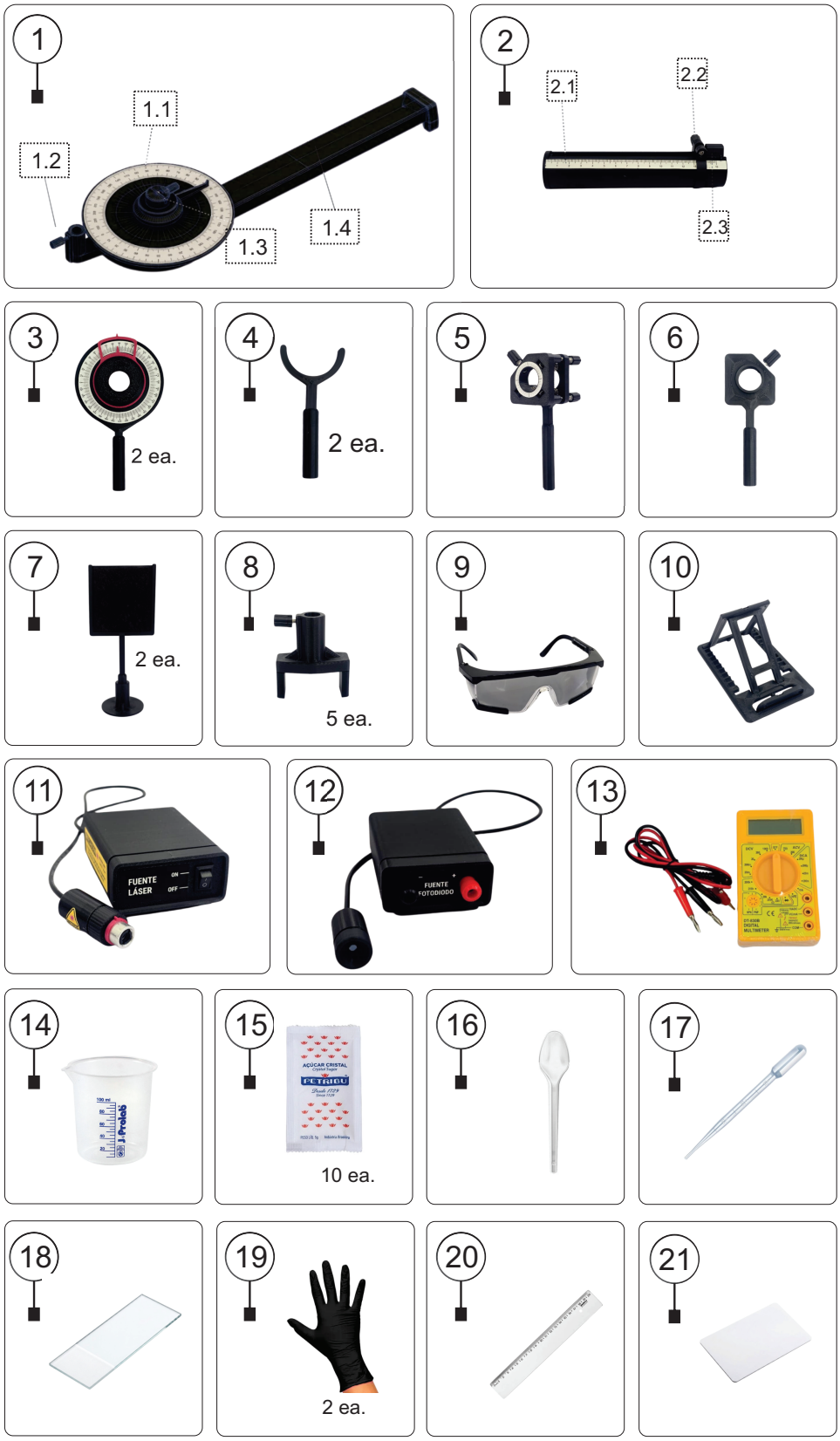
Checking and Identifying the Kit

The photograph below gives an overview of the kit materials. The diagram on the next page identifies each component. Check the items before starting assembly, and report any missing or damaged item to the staff. List numbers refer exclusively to the numbers in the diagram.



Overview of the components supplied in the experimental kit.

Kit Identification Diagram



Item	Description	Quantity
1	Optical rail and goniometer assembly with sample holder	1
1.1	Goniometer angular scale	—
1.2	Photodiode mount	—
1.3	Sample holder	—
1.4	Metal rail	—
2	Variable-length cell	1
2.1	External millimetre scale	—
2.2	Sealing ring	—
2.3	Cylindrical piston	—
3	Polarizers with angular scale	2
4	Cell mounts	2
5	Laser mount with adjustment screws	1
6	Photodiode mount	1
7	Screens to block laser reflections	2
8	Bases for support rods	5
9	Protective goggles	1
10	Multimeter mount	1
11	Power supply and laser emitter	1 set
12	Photodiode power supply and sensor	1 set
13	Multimeter with test leads	1 set
14	Graduated beaker	1
15	Sugar sachets	10
16	Plastic spoon	1
17	Graduated transfer pipette	1
18	Glass plate	1
19	Surgical gloves	2
20	Millimetre ruler	1
21	White card for viewing the laser beam	1

The quantities reproduce the illustrated materials and the labels in the diagram. A dash in the quantity column denotes a component of assembly 1 or 2, not an additional piece.

Q1 — Rotation of the polarization direction in a sugar solution (10 points)

Keep the laser on the right and the first polarizer fixed throughout all series. Before recording data, align the beam with the centres of the cell, analyser and photodiode, and determine the angular resolution. These adjustments are operating procedures, not scored questions.

PART A — Reference curve with the empty cell (4,0 pts)

Install the empty cell, keep the scale zeros fixed, and rotate only the analyser.

A.1 **2,0 pts**

Guided background. The empty-cell curve includes the optical windows and provides the reference for all measurements with solution.

Task. Measure the reference current i_{ref} as a function of the angular position θ_A. Present a table of θ_A and i_{ref}, including units and the applicable instrumental uncertainties. Plot i_{ref} against θ_A, identifying the axes and showing uncertainties where appropriate.

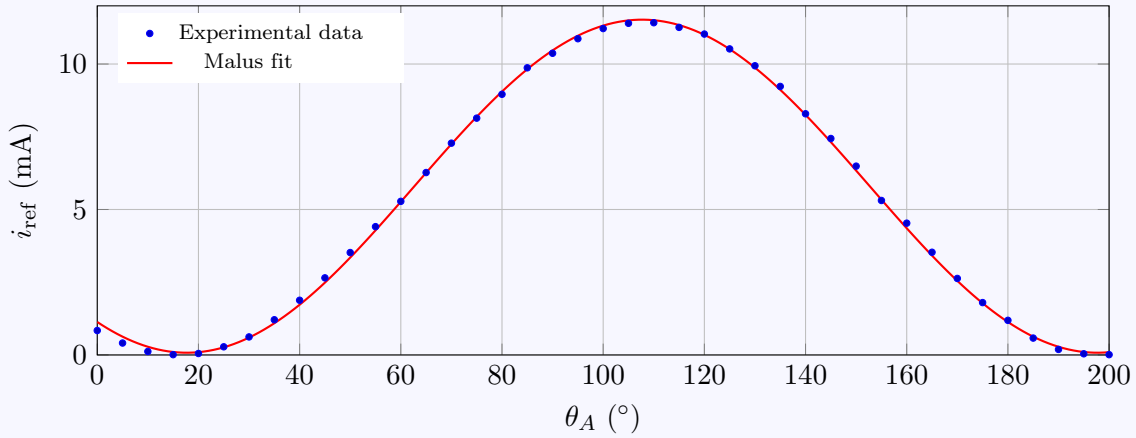
Illustrative solution and required data.

There were 41 angular positions from 0° to 200°, in steps of 5°. The angular resolution was 1°; half the smallest division, $\Delta\theta = 0{,}5°$, was used as the reading limit. Each current was measured once in the 20 mA range, for which $D = 0{,}01$ mA. Hence,

$$\Delta i_{\text{inst}} = 0{,}010|i| + 0{,}02\,\text{mA}.$$

No statistical uncertainty of the mean was calculated, since each cell contains a single reading. Instrumental limits were rounded upward to 0,01 mA.

θ_A (°)	i_{ref} (mA)	Δi (mA)	θ_A (°)	i_{ref} (mA)	Δi (mA)
0	0,84	0,03	105	11,40	0,14
5	0,41	0,03	110	11,42	0,14
10	0,12	0,03	115	11,26	0,14
15	0,01	0,03	120	11,03	0,14
20	0,05	0,03	125	10,52	0,13
25	0,28	0,03	130	9,94	0,12
30	0,62	0,03	135	9,23	0,12
35	1,21	0,04	140	8,29	0,11
40	1,88	0,04	145	7,44	0,10
45	2,65	0,05	150	6,49	0,09
50	3,52	0,06	155	5,31	0,08
55	4,41	0,07	160	4,53	0,07
60	5,28	0,08	165	3,53	0,06
65	6,27	0,09	170	2,63	0,05
70	7,28	0,10	175	1,80	0,04
75	8,14	0,11	180	1,19	0,04
80	8,96	0,11	185	0,58	0,03
85	9,87	0,12	190	0,19	0,03
90	10,37	0,13	195	0,04	0,03
95	10,87	0,13	200	0,01	0,03
100	11,22	0,14			



The curve shows the periodic behaviour expected from Malus's law, with a minimum near 15°–20° and a maximum near 108°.

Note. The Malus-law fit is included in Q1–A.1 only to make the data distribution easier to visualize. A fit is not required in
Q1–A.1.

A.2 **2,0 pts**

Guided background. The angular position of the reference curve can be described by the phase parameter ϕ_{ref} (polarization direction) in a shifted Malus law.

Task. Fit the data from A.1 to the model

$$i_{\text{ref}}(\theta_A) = a_{\text{ref}} + b_{\text{ref}} \cos^2(\theta_A - \phi_{\text{ref}})$$

and determine ϕ_{ref} with its uncertainty. State the fitting method used and report the fitted parameters with their units.

Illustrative solution and required data.

The problem can be solved by a sequence of simple linear regressions. First estimate ϕ_{ref} graphically from the maximum position. Although the largest measured point is at 110°, the smoothed distribution allows the initial estimate $\phi = 108°$. For a candidate value of ϕ, calculate

$$x_k = \cos^2(\theta_{A,k} - \phi),$$

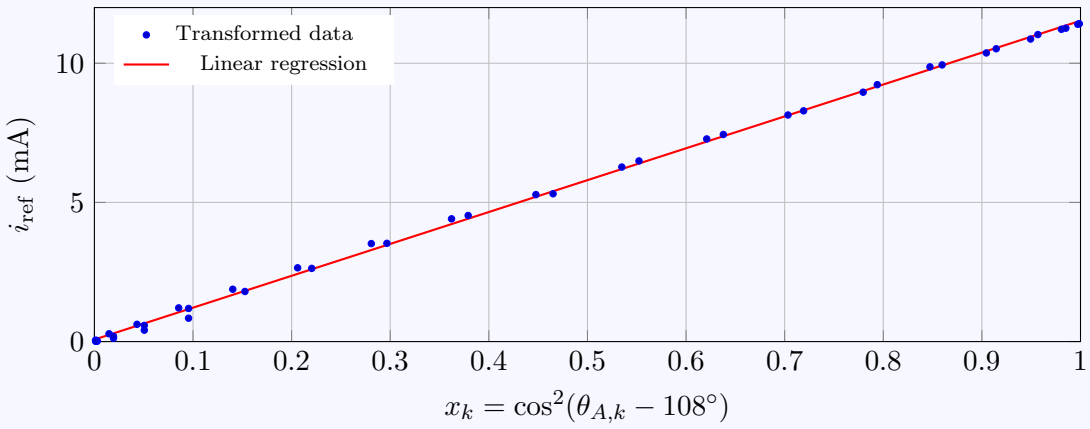
and perform the linear regression

$$i_{\text{ref},k} = a_{\text{ref}} + b_{\text{ref}} x_k.$$

The initial estimate $\phi = 108°$ gives

$$a_{\text{ref}} = (0{,}072 \pm 0{,}029)\,\text{mA}, \qquad b_{\text{ref}} = (11{,}454 \pm 0{,}051)\,\text{mA},$$

with $R^2 = 0{,}99922$.



The small systematic deviation from the straight line shows that 108° is still an initial phase estimate. To improve it, repeat the linear regression for values near 108° and choose the one maximizing R^2 or, equivalently, minimizing the sum of squared residuals. The maximum occurs at

$$\phi_{\text{ref}} = 107{,}687°.$$

The corresponding linear regression gives

$$\boxed{a_{\text{ref}} = (0{,}077 \pm 0{,}027)\,\text{mA}}, \qquad \boxed{b_{\text{ref}} = (11{,}449 \pm 0{,}048)\,\text{mA}},$$

with $R^2 = 0{,}99932$. The curvature of the residual sum near the minimum gives a phase-fit uncertainty of $0{,}131°$. Combining it with the $0{,}5°$ contribution from angular resolution,

$$u_{\phi,\text{ref}} = \sqrt{(0{,}131°)^2 + (0{,}5°)^2} = 0{,}517°.$$

Therefore,

$$\boxed{\phi_{\text{ref}} = (107{,}7 \pm 0{,}5)°}.$$

This procedure avoids simultaneous nonlinear regression: each calculation is a linear regression of i_{ref} against $\cos^2(\theta_A - \phi)$, combined with a one-dimensional phase search.

PART B — Rotation of light polarization on passing through a sugar solution (6,0 pts)

Rotation of the polarization direction shifts the intensity curve horizontally. Choose a reference point on the curve and check how it shifts with the path length L travelled by light in the sample.

B.1 **2,0 pts**

Guided background. Comparisons require the geometry and angular references to remain unchanged between series.

Task. Prepare a solution containing 0,40 g of sugar per millilitre of water, measured before dissolution. Choose at least three different lengths $L_1, \ldots, L_n$ within the cell's length limit. Choose a reference point on the intensity curve and construct a table of L as a function of the curve shift $\Delta\phi$.

Illustrative solution and required data.

The nominal concentration is

$$C = 0{,}40\,\text{g}\,\text{mL}^{-1}.$$

Only ten 5 g sugar sachets were supplied, so the student had to devise a preparation sufficient for all series, without preparing a different solution for each length. The cell allowed a maximum length of 15 cm; its geometry suggested an internal diameter between 2 cm and 3 cm. Taking $d = 2{,}5$ cm for the calculation, its maximum usable volume, approximated as a cylinder, is

$$V_{\text{máx}} = \pi \left(\frac{d}{2}\right)^2 L_{\text{máx}} = \pi(1{,}25\,\text{cm})^2(15\,\text{cm}) \approx 73{,}6\,\text{cm}^3 = 73{,}6\,\text{mL}.$$

The estimated geometric volume shows that more than approximately 74 mL of solution must be prepared to fill the cell to its maximum length and leave a margin for transfers and successive series. Since the required concentration is defined using the *water volume measured before*

dissolution, a simple, exact preparation uses eight sachets:

$$m = 8(5\,\mathrm{g}) = 40\,\mathrm{g}, \qquad V_{\mathrm{agua}} = 100\,\mathrm{mL}.$$

Indeed,

$$C = \frac{m}{V_{\mathrm{agua}}} = \frac{40\,\mathrm{g}}{100\,\mathrm{mL}} = 0{,}40\,\mathrm{g\,mL^{-1}}.$$

The 100 mL is the initial water volume, not the final solution volume after adding sugar. This preparation exceeds the cell's estimated maximum usable volume, allows the same solution to be reused for all length series, and leaves two sachets in reserve.

Use as the reference the value previously determined by the procedure in Q1–A.2,

$$\phi_{\mathrm{ref}} = 108^\circ.$$

This value is not obtained by manually locating the maximum with the solution samples. For each length $L_k > 0$, rotate the analyser slowly while watching the multimeter current. Identify ϕ_k as the angle at the current maximum and record it directly with the angular scale's resolution. The corresponding rotation is calculated from

$$\Delta\phi_k = \phi_k - \phi_{\mathrm{ref}}.$$

For angular differences to be comparable, retain the reference-measurement conditions: assembly alignment and geometry, optical-element and detector positions, input-polarizer orientation, the multimeter's 20 mA range, and lighting conditions. Between solution series, change only the length L_k traversed by the beam.

The analyser's angular resolution is 1°, so use half a division as the uncertainty of each directly measured maximum position:

$$u_{\phi_k} = \frac{1^\circ}{2} = 0{,}5^\circ.$$

The result of Q1–A.2 also gives $u_{\phi,\mathrm{ref}} = 0{,}5^\circ$. Each solution maximum is a single reading, so there is no statistical repeatability contribution. Assuming independent uncertainties in ϕ_k and ϕ_{ref}, propagate the difference

$$\Delta\phi_k = \phi_k - \phi_{\mathrm{ref}},$$

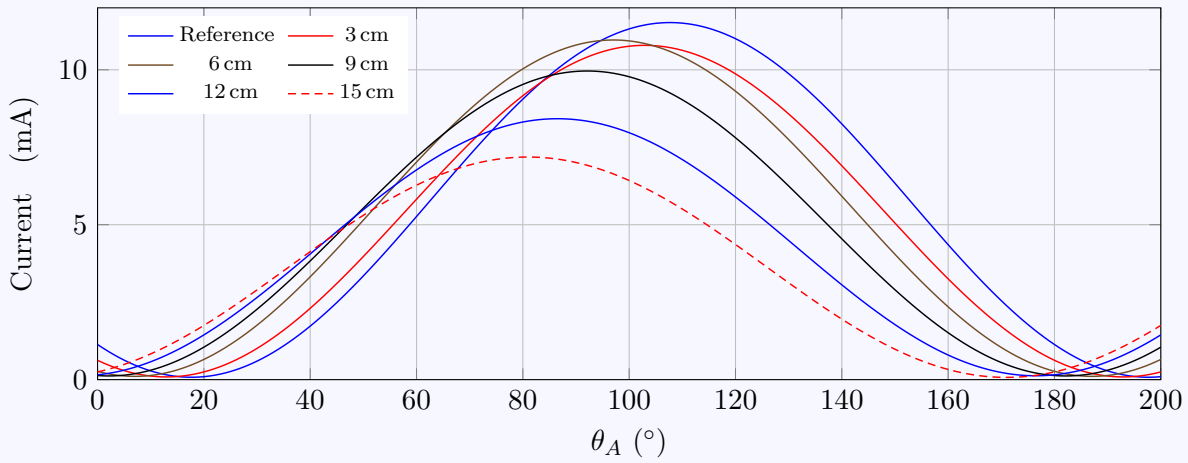
by adding their variances:

$$u_{\Delta\phi} = \sqrt{u_{\phi,k}^2 + u_{\phi,\mathrm{ref}}^2} = \sqrt{(0{,}5^\circ)^2 + (0{,}5^\circ)^2} = 0{,}707^\circ \approx 0{,}7^\circ.$$

L (cm)	ϕ (°)	u_ϕ (°)	$\Delta\phi$ (°)
0	108	0,5	0,0
3	103	0,5	$-5,0 \pm 0,7$
6	97	0,5	$-11,0 \pm 0,7$
9	92	0,5	$-16,0 \pm 0,7$
12	87	0,5	$-21,0 \pm 0,7$
15	81	0,5	$-27,0 \pm 0,7$

In the first row, $\Delta\phi = 0{,}0°$ by definition, since that reading establishes the reference. With the angular convention used, the negative sign means that the polarization rotates counterclockwise as L increases.

Current-versus-angle graphs document the experimental results and help visualize their shifts. However, they are not a scored requirement in B.1: including them earns no extra points, and omitting them incurs no penalty.



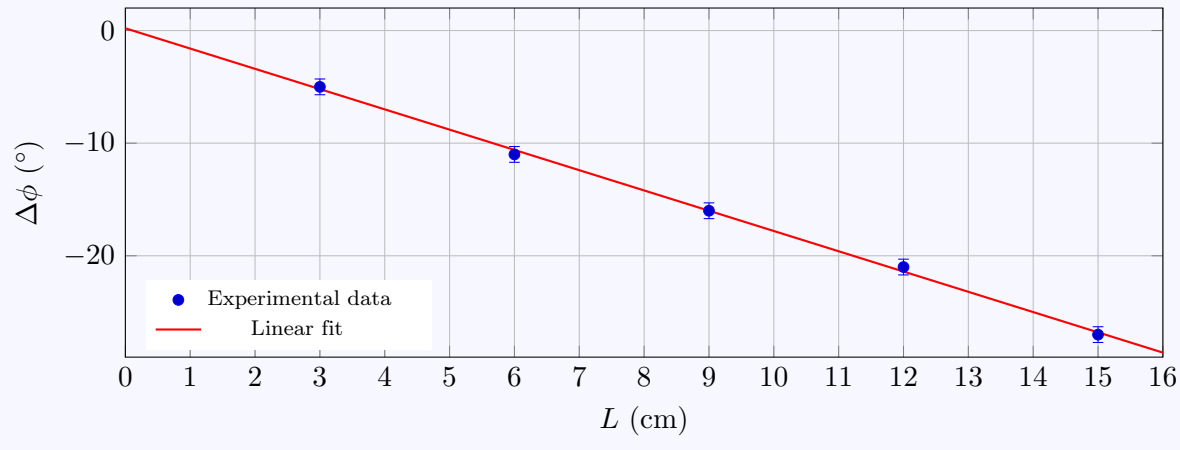
B.2 **2,0 pts**

Guided background. Optical rotation is determined from the difference between the angular position of each sample's maximum and that of the reference maximum.

Task. Plot $\Delta\phi(L_k)$ against L_k, with the corresponding error bars.

Illustrative solution and required data.

The graph shows an approximately linear dependence of $\Delta\phi$ on L. The vertical bars represent $u_{\Delta\phi}$. Horizontal bars are omitted because the resolution used to set the cell lengths was not reported.



The linear fit with a free intercept was

$$\Delta\phi = (0{,}20 \pm 0{,}73)^\circ - (1{,}800 \pm 0{,}074)\,\frac{^\circ}{\text{cm}}L, \qquad R^2 = 0{,}99863.$$

The intercept agrees with zero. Forcing the line through the origin gives the slope

$$m = -(1{,}782 \pm 0{,}031)\,\frac{^\circ}{\text{cm}}.$$

B.3 **2,0 pts**

Guided background. The dependence to be tested is

$$\Delta\phi(L) = \alpha(\lambda, T)\, L^n C,$$

where $\alpha(\lambda, T)$ characterizes the medium and depends on the wavelength λ and temperature T; n is the length exponent, and C is the solution concentration. During the experiment, λ, T and C must be treated as constant.

Task. From the behaviour observed in the graph in B2, propose a suitable fit and determine n and α.

Illustrative solution and required data.

Since C, λ and T remain constant, the proposed dependence has only two unknown parameters: the exponent n and coefficient α. The point $L = 0$ confirms that rotation vanishes without a sample, but is excluded from the power-law fit because it provides no information for determining n.

Fit the five points with $L > 0$ directly to the model

$$\Delta\phi = \alpha C L^n$$

by weighted least squares. Assign each point the weight $w_k = 1/u^2_{\Delta\phi,k}$ and minimize

$$\chi^2(\alpha, n) = \sum_k \frac{[\Delta\phi_k - \alpha C L_k^n]^2}{u^2_{\Delta\phi,k}}.$$

An alternative way to visualize the procedure is to take absolute values and write

$$\ln\left(\frac{|\Delta\phi|}{C}\right) = \ln|\alpha| + n \ln L.$$

On a plot of $\ln(|\Delta\phi|/C)$ against $\ln L$, the slope is n and the intercept is $\ln|\alpha|$. Direct fitting is preferable for the final result because it uses uncertainties in the original angular variable and avoids the distortion introduced by taking logarithms.

Using $C = 0{,}40\,\mathrm{g\,mL^{-1}}$ and $u_{\Delta\phi} = 0{,}7°$ gives

$$\boxed{n = 1{,}01 \pm 0{,}05},$$

$$\boxed{\alpha = (-4{,}32 \pm 0{,}59)\,°\,\mathrm{mL\,g^{-1}\,cm^{-n}}}.$$

The fit gives $R^2 = 0{,}99859$. The deviation from the expected linear exponent is

$$\frac{|n-1|}{u_n} = \frac{|1{,}01-1|}{0{,}05} = 0{,}2,$$

far below one standard uncertainty. The data therefore agree with $n = 1$ and confirm, within experimental precision, that rotation is proportional to the path length in the solution.

The negative sign of α retains the table's convention: it denotes counterclockwise rotation. Its magnitude is $|\alpha| = (4{,}32 \pm 0{,}59)$ in the stated units. Since the dimensions of α depend on the fitted exponent, they are initially written with $\mathrm{cm^{-n}}$.

Since n agrees with 1, the linear physical model may be imposed:

$$\Delta\phi = \alpha C L.$$

The slope of the line forced through the origin in B.2 satisfies $m = \alpha C$. Hence,

$$\alpha = \frac{m}{C} = \frac{-1{,}782}{0{,}40} = -(4{,}45 \pm 0{,}08)\,°\,\mathrm{mL\,g^{-1}\,cm^{-1}}.$$

Here the uncertainty of the prepared concentration is taken as negligible. Thus,

$$u_\alpha = \frac{u_m}{C} = \frac{0{,}031}{0{,}40} = 0{,}078 \approx 0{,}08\,°\,\mathrm{mL\,g^{-1}\,cm^{-1}}.$$

If a concentration uncertainty u_C were available, it should be included using

$$\left(\frac{u_\alpha}{|\alpha|}\right)^2 = \left(\frac{u_m}{|m|}\right)^2 + \left(\frac{u_C}{C}\right)^2.$$

The results with a free n and with $n = 1$ agree within their uncertainties. The second is more precise because it incorporates the linear dependence supported by the data.

Q2 — Polarization by reflection from a glass plate (10 points)

Unscored operating adjustments

Assemble the system with the laser on the right. Without the plate, align the rail, goniometer centre and photodiode for direct beam incidence. Install the plate with its normal at $\theta_i = 0°$ and determine the angular resolution. Retain this reference throughout Q2.

PART A — Intensity of reflected light (6,0 pts)

Before the angular scan, identify the operating orientations for s and p polarization and balance the reflected intensities. After balancing, keep the laser orientation fixed and retain the established angular references.

A.1 **2,0 pts**

Guided background. With the plate at $\theta_i \approx 70°$, the maximum reflected intensity as the polarizer rotates defines α_s, the operating reference for s polarization. The observed minimum defines α_p, the operating reference for p polarization, provided its separation from α_s agrees with 90° within the uncertainties.

Task. Rotate the polarizer through 180° and determine α_s and α_p, their uncertainties and the corresponding reflected intensities. Check that $|\alpha_p - \alpha_s|$ agrees with 90°. If not, switch off the laser, rotate its body slightly about its longitudinal axis, recheck alignment, and repeat. Tabulate the tested laser orientations, α_s, and α_p, currents and angular separation.

Illustrative solution and required data.

The position of maximum current identifies α_s, and the minimum identifies α_p. For angular resolution 1°, assign half a division as the uncertainty of each reading:

$$\alpha_s = (175{,}0 \pm 0{,}5)°, \qquad \alpha_p = (85{,}0 \pm 0{,}5)°.$$

The angular separation is

$$\Delta\alpha = |\alpha_p - \alpha_s| = |85{,}0° - 175{,}0°| = 90{,}0°,$$

with uncertainty

$$u_{\Delta\alpha} = \sqrt{u_{\alpha_p}^2 + u_{\alpha_s}^2} = \sqrt{(0{,}5°)^2 + (0{,}5°)^2} \approx 0{,}7°.$$

Therefore,

$$\boxed{\Delta\alpha = (90{,}0 \pm 0{,}7)°},$$

so the separation fully agrees with orthogonal s and p polarizations.

Trial	Relative laser rotation	α_s	i_s (mA)	α_p	i_p (mA)	$\lvert\alpha_p - \alpha_s\rvert$	Compatible
1	0°	170°	$2{,}42 \pm 0{,}05$	65°	$1{,}05 \pm 0{,}04$	105°	No
2	5°	175°	$2{,}10 \pm 0{,}05$	75°	$1{,}28 \pm 0{,}04$	100°	No
Final	10°	175°	$1{,}65 \pm 0{,}04$	85°	$1{,}70 \pm 0{,}04$	90°	Yes

For every angle use $u_\alpha = 0{,}5°$ and, by propagation, $u_{\Delta\alpha} = 0{,}7°$. Thus the first two trial separations differ from 90° by more than 0,7°, whereas the final adjustment agrees with orthogonality. A relative rotation of 5° of the laser body is used between successive configurations.

Note. The first two trials are illustrative, showing how to document progressive correction of the laser orientation. The supplied data apply only to the final adjustment:
$\alpha_s = 175°$, $\alpha_p = 85°$, $i_s = 1{,}65\,\text{mA}$ and $i_p = 1{,}70\,\text{mA}$.

A.2 **2,0 pts**

Guided background. Balancing establishes a fixed source orientation for comparing the two polarizations. With the plate at $\theta_i = 70°$, compare the currents $i_{s,70}$ and $i_{p,70}$ measured at the fixed positions α_s and α_p.

Task. Alternate the polarizer between α_s and α_p and record $i_{s,70}$ and $i_{p,70}$. If necessary, switch off the laser, rotate its body slightly, and repeat the readings without redefining α_s or α_p. Calculate

$$\varepsilon_{\text{bal}} = \frac{|i_{s,70} - i_{p,70}|}{(i_{s,70} + i_{p,70})/2}$$

and adjust the source until $\varepsilon_{\text{bal}} \leq 0{,}10$. Report the readings for every attempt, the final laser orientation and the percentage difference $100\varepsilon_{\text{bal}}\,\%$.

Illustrative solution and required data.

The supplied readings for the final configuration are

$$i_{s,70} = (1{,}65 \pm 0{,}04)\,\text{mA}, \qquad i_{p,70} = (1{,}70 \pm 0{,}04)\,\text{mA}.$$

Instrumental limits were calculated for the 20 mA range, for which $D = 0{,}01\,\text{mA}$ and $\Delta i = 0{,}010|i| + 0{,}02\,\text{mA}$. Hence,

$$\varepsilon_{\text{bal}} = \frac{|1{,}65 - 1{,}70|}{(1{,}65 + 1{,}70)/2} = 0{,}0299.$$

Therefore,

$$100\varepsilon_{\text{bal}}\,\% = 2{,}99\,\% < 10\,\%,$$

and the final balance is acceptable.

The relative rotation of 10° in the A.1 table belongs to the illustrative sequence; it is not an absolute experimental reading of the laser body. The supplied data verify only the final configuration's balance.

Choose incidence angles spanning both sides of the current minimum for p polarization. Use smaller steps near the minimum. At each angle, place the photodiode in the specular direction and measure i_s and i_p without changing the laser orientation.

A.3 **1,0 pts**

Guided background. Use the same angular scale for both polarization curves so that they can be compared directly.

Task. Present a table of θ_i, i_s, i_p, units and uncertainties. Plot $i_s(\theta_i)$ and $i_p(\theta_i)$ together on the same sheet of graph paper, using different symbols or line styles and a legend to distinguish them.

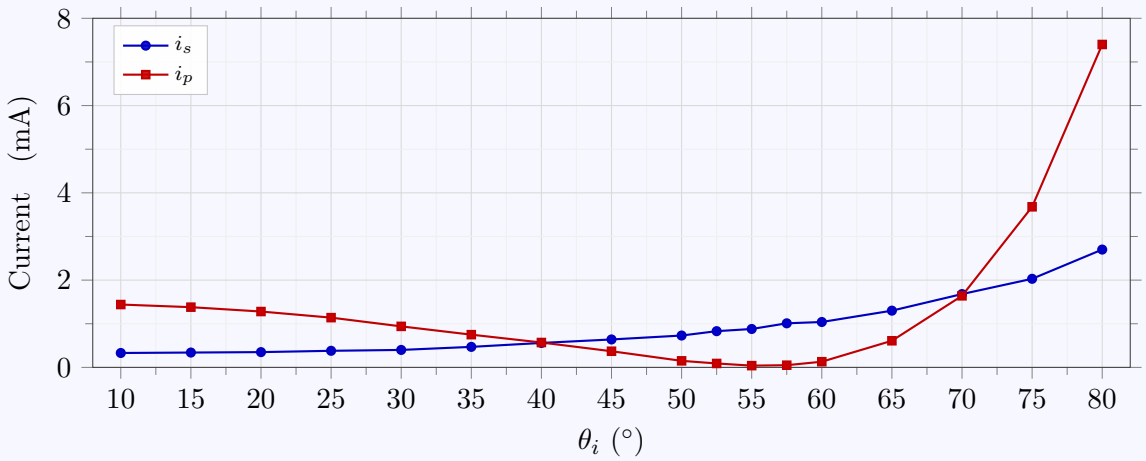
Illustrative solution and required data.

The angular resolution is the same as in Q1, $\Delta\theta_i = 0{,}5°$, and every current was measured once in the 20 mA range. For each reading, use

$$\Delta i = 0{,}010|i| + 0{,}02\,\text{mA},$$

rounding the limit upward to 0,01 mA.

θ_i (°)	i_s (mA)	Δi_s (mA)	i_p (mA)	Δi_p (mA)
10	0,33	0,03	1,44	0,04
15	0,34	0,03	1,38	0,04
20	0,35	0,03	1,28	0,04
25	0,38	0,03	1,14	0,04
30	0,40	0,03	0,94	0,03
35	0,47	0,03	0,75	0,03
40	0,56	0,03	0,57	0,03
45	0,64	0,03	0,37	0,03
50	0,73	0,03	0,15	0,03
52,5	0,83	0,03	0,09	0,03
55	0,88	0,03	0,04	0,03
57,5	1,01	0,04	0,05	0,03
60	1,04	0,04	0,13	0,03
65	1,30	0,04	0,61	0,03
70	1,68	0,04	1,64	0,04
75	2,03	0,05	3,68	0,06
80	2,70	0,05	7,40	0,10



The p component has a pronounced minimum between 55° and 57,5°, whereas the s component's current increases without vanishing in that region. This solution uses $i_p(52{,}5°) = 0{,}09\,\text{mA}$ and $i_p(57{,}5°) = 0{,}05\,\text{mA}$ to show the minimum near 56° more clearly.

A.4 **1,0 pt**

Guided background. The ratio $q = i_s/i_p$ is defined only when i_p is positive and distinguishable from zero within its uncertainty.

Task. Calculate q for each valid angle, add the values to a table, and plot $q(\theta_i)$. Do not calculate or plot q at points whose denominator is not positive and distinguishable from zero within its uncertainty.

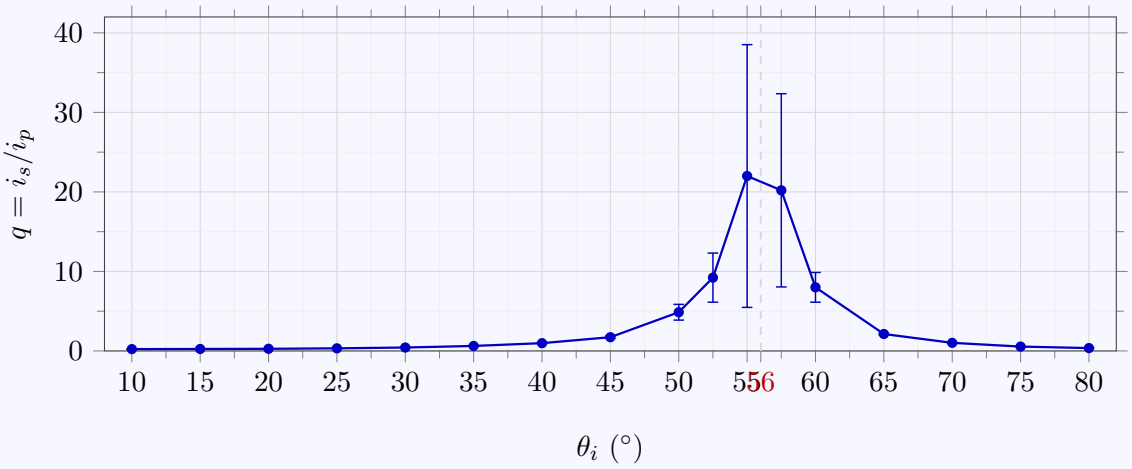
Illustrative solution and required data.

Calculate

$$q = \frac{i_s}{i_p}, \qquad u_q = q\sqrt{\left(\frac{u_{i_s}}{i_s}\right)^2 + \left(\frac{u_{i_p}}{i_p}\right)^2}.$$

All points are valid because $i_p - \Delta i_p > 0$.

θ_i (°)	$q = i_s/i_p$	Δq
10	0,23	0,02
15	0,25	0,02
20	0,27	0,02
25	0,33	0,03
30	0,43	0,03
35	0,63	0,05
40	0,98	0,07
45	1,73	0,16
50	4,87	0,99
52,5	9,22	3,09
55	22,00	16,52
57,5	20,20	12,15
60	8,00	1,87
65	2,13	0,12
70	1,02	0,03
75	0,55	0,02
80	0,36	0,01



The large values of q at 55° and 57,5° directly suggest a maximum around

$$\boxed{\theta_i \approx 56^\circ}.$$

PART B — Brewster angle and refractive index (4,0 pts)

B.1 **2,0 pts**

Guided background. The Brewster angle corresponds to the minimum of i_p and the region of the largest reliable ratio q.

Task. Determine θ_B by comparing the minimum of i_p with the region of the largest reliable ratio. Calculate $\theta_B^{\text{exp}} \pm u_\theta$ and show how the position and uncertainty follow from the angular resolution and local sampling.

Illustrative solution and required data.

The minimum of i_p and maximum of q lie between 55° and 57,5°. Adopt directly

$$\theta_B^{\text{exp}} = 56{,}0^\circ.$$

With angular resolution 1° and local sampling step 2,5°,

$$u_\theta = \sqrt{\left(\frac{2{,}5^\circ}{2}\right)^2 + \left(\frac{1{,}0^\circ}{2}\right)^2} \approx 1{,}3^\circ.$$

Therefore,

$$\boxed{\theta_B^{\text{exp}} = (56{,}0 \pm 1{,}3)^\circ}.$$

B.2 **1,0 pts**

Guided background. For incidence from air into glass, $n_{\text{vidrio}} = n_{\text{aire}} \tan\theta_B$.

Task. Use $n_{\text{aire}} = 1{,}000$ and calculate n_{vidrio} with its uncertainty.

Illustrative solution and required data.

With $n_{\text{aire}} = 1{,}000$,

$$n_{\text{vidrio}} = 1{,}000 \tan 56{,}0^\circ = 1{,}483 \approx 1{,}48.$$

$$u_n = 1{,}000 \sec^2 56{,}0^\circ \left(1{,}3\frac{\pi}{180}\right) \approx 0{,}07.$$

$$\boxed{n_{\text{vidrio}} = 1{,}48 \pm 0{,}07}.$$

The limits are

$$n_- = \tan 54{,}7^\circ = 1{,}41, \qquad n_+ = \tan 57{,}3^\circ = 1{,}56.$$

B.3 **1,0 pts**

Guided background. The nominal value $n = 1{,}50$ is used only as a reference for comparison.

Task. Calculate the percentage deviation between θ_B^{exp} and θ_B^{teo}. Present a final table containing the angular resolution, θ_B^{exp}, u_θ, θ_B^{teo}, n_{vidrio}, its uncertainty limits and the percentage deviation.

Illustrative solution and required data.

The theoretical reference value is

$$\theta_B^{\text{teo}} = \arctan\left(\frac{1{,}50}{1{,}000}\right) = 56{,}31^\circ.$$

The percentage deviation is

$$\text{deviation} = 100\,\frac{|56{,}0^\circ - 56{,}31^\circ|}{56{,}31^\circ} = 0{,}55\,\%.$$

Quantity	Result
Angular resolution	1°
θ_B^{exp}	$56{,}0^\circ$
u_θ	$1{,}3^\circ$
θ_B^{teo}	$56{,}31^\circ$
n_{vidrio}	$1{,}48 \pm 0{,}07$
Limits of n	$[1{,}41;\ 1{,}56]$
Angular deviation	$0{,}55\,\%$

The experimental values agree with the nominal references $\theta_B^{\text{teo}} = 56{,}31^\circ$ and $n = 1{,}50$.

Glossary

Analyser
A polarizer placed after the sample or interface, used to investigate the direction or state of polarization of the emerging light.

Brewster angle, θ_B
The incidence angle at which the reflectance of the p component is minimal and, ideally, zero at an interface between transparent media.

Optical activity
A medium's ability to rotate the polarization direction of linearly polarized light as it propagates.

Sample length, L
The geometric distance travelled by the beam inside the solution, with all other experimental conditions fixed. In Q1, the lengths freely chosen by the student are denoted $L_1, \ldots, L_n$, where n is the number of lengths chosen within the cell's dimensional limit. In a homogeneous medium of refractive index n_{medio}, the corresponding optical path is $n_{\text{medio}}L$.

Background current, i_{fondo}
Current recorded by the detection system without the useful light signal, under the same conditions and in the same range as the measurements. It may include detector dark current, electronic contributions and residual ambient light.

Standard deviation of the mean
Estimate of the mean's statistical spread for N repeated readings, $s_{\bar{x}} = s/\sqrt{N}$, where s is the sample standard deviation.

Transmission axis
The direction of a polarizer along which the electric-field component is preferentially transmitted.

Enantiomers
Molecular structures that are non-superimposable mirror images of one another.

Laser
A source producing an intense, directional, approximately monochromatic beam with high coherence.

Malus's law
The relation describing the intensity transmitted by an analyser in terms of the squared cosine of the angle between its transmission axis and the incident polarization.

Instrumental limit
The error limit calculated from the instrument's accuracy specification. It is not, by itself, a standard deviation or the statistical uncertainty of the mean.

Linearly polarized light
Light whose electric field oscillates along a well-defined transverse direction.

Unpolarized light
Light whose electric-field oscillations have no preferred transverse direction over time.

Cell adjustment knob
A manually operated part that tightens the sealing-ring screw and prevents leaks. It is not the cursor used to read the length L.

Chiral molecule
A molecule whose structure cannot be superimposed on its mirror image by rotations or translations.

Normalization
Division of the values in a series by a reference. In Q1, use $i^{\mathrm{corr}}_{\mathrm{máx},k}$, the largest corrected current among the regular-grid points selected for series k; this need not coincide with the maximum of the complete curve. In Q2, calculate the ratio of the two reflected-polarization currents, subtracting background current when necessary, without normalizing each curve by its own maximum. This ratio is not a direct measure of reflectance.

Plane of incidence
The plane defined by the incident ray and the surface normal at the point of incidence.

p polarization
Polarization whose electric field oscillates parallel to the plane of incidence.

s polarization
Polarization whose electric field oscillates perpendicular to the plane of incidence.

Circular polarization
A state in which the tip of the electric-field vector rotates around the propagation direction while its magnitude remains approximately constant.

Reflectance
The ratio of reflected light power to incident light power for a specified polarization and geometry.

Optical rotation
Rotation of the polarization direction during propagation in an optically active medium; in this examination, a sugar solution.

Mechanical zero of the goniometer
The fixed mechanical reference of the assembly, with the rail axis, goniometer rotation centre and photodiode centre aligned for direct incidence without the plate. With the plate installed and its normal aligned with the beam, this reference corresponds to $\theta_i = 0^\circ$ and must not be redefined from current readings.